MODULE FOUR

Patterns, Functions and Algebra

MODULE FOUR PATTERNS, FUNCTIONS AND ALGEBRA

Contents

UNIT 1

Patterns

Trainer's Note:
Unit 1 introduces the learner to two kinds of patterns:
Geometric patterns: *These are patterns made up of geometric shapes. To create and extend these patterns we move the shapes that make up the pattern. We hope that the learners can see these patterns quite easily. You may need to provide support – this could be done by referring learners to patterns in floor tiles, patterns on bathroom walls etc. The rest of the module deals only with number patterns and if learners struggle a great deal with geometric patterns it is possibly best to move on.*
Number patterns: *Before working through the number pattern activities you may want to play a game of guess-the-next-number. In this game you think of a rule for making a number pattern, give the learners the first terms in the pattern and let them guess the next two or more numbers. Both the learners who get the answer correct and those who don't should be asked to explain how they came to the numbers that they did. Talking about their thinking can help learners to "get into" the activities.*

In this unit you will address the following:

Unit Standard 7448
SO1:
Recognize, identify and describe patterns in various contexts. (numeric, geometric, patterns from a variety of contexts.)

SO2:
Complete, extend and generate patterns in a variety of contexts. (numeric, geometric, patterns from a variety of contexts.)

Unit Standard 7464
SO1:
Identify geometric shapes and patterns in cultural products. (shapes of and decorations on cultural products such as drums, pots, mats, buildings, and necklaces.)

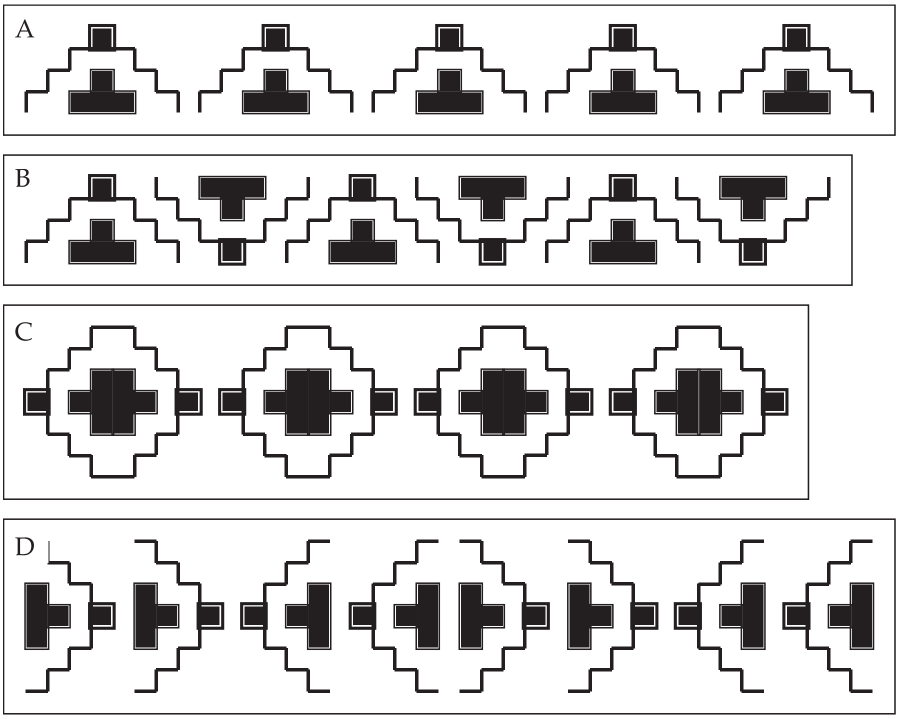
SO2:
Analyze similarities & differences in shapes & patterns, & effect of colour, used by cultures. (analyze similarities and differences in shapes and patterns, and the effect of colour, used by different cultures.)

To do this you will:
- distinguish between geometric and numeric patterns;
- investigate and analyse both geometric and numeric patterns;
- explain and justify patterns observed in both geometric and numeric patterns;
- complete tables of values for numeric patterns;
- use tables of value for numeric patterns to identify rules used to generate numeric patterns;
- answer questions based on information derived from tables of values.

1. Geometric patterns

A pattern consists of objects arranged in order according to a rule.

Below are several illustrations of African wall patterns. You will almost certainly have seen such patterns in homes, sometimes in the tiles and sometimes in the paintwork. You may even have made such a pattern on the walls of your crèche.



Time needed
15 minutes

Activity 1: Investigating geometric patterns

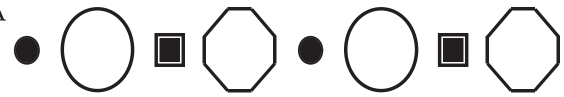
Work alone

Look at the patterns above again. Answer these questions about the patterns.

1. Draw a sketch of the object that has been used as the basis for each of these patterns.
2. For each pattern, circle the basic shape then describe, in words, how the shape was moved to create the pattern. Describe both the direction and size of the move. Your description of how to move the shape is called the rule that generates the pattern.
3. Draw a sketch of at least one other pattern that can be generated using the same shape. Describe the rule you used to generate the pattern.

The patterns below have been generated using more than one shape.

A



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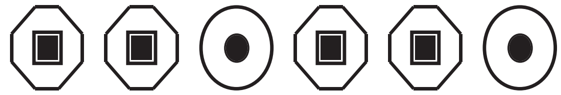

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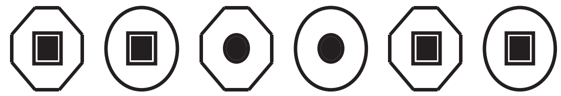
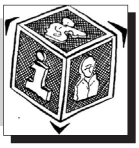


1. For each of the above patterns draw the next four shapes in the sequence.
2. For each pattern describe, in words, the rule that you used to work out the next four shapes in the sequence.
3. Draw a sketch of at least three other patterns that can be generated using the same objects. Describe the rule you used to generate the patterns.



2. Number patterns

The sequence of numbers: 4 9 19 39 can be thought of as a number pattern.

The main difference between number patterns and geometric patterns is that number patterns are made up of numbers and not shapes.

In number patterns the rule which makes the sequence of numbers consist of mathematical operations or calculations that are repeated. In geometric patterns the rule which makes the pattern is a sequence of moves that is repeated.

4	→	9	→	19	→	39	→
Starting number (1st term)	Multiply the number by 2 and add 1	Second number (2nd term)	Multiply the number by 2 and add 1	Third number (3rd term)	Multiply the number by 2 and add 1	Fourth number (4th term)	

In the example above the rule is "multiply the number by 2 and add 1." The number pattern that results is:

4 9 19 38 … … … …

We say that the number 19 is the third term in the pattern.

Time needed
20 minutes

Activity 2:
Extending and generating number patterns

Work alone

1. Write down the next four terms in the pattern above.
2. Work out the number pattern that uses the same rule but starts with 6 as the first number.
3. Complete the number patterns below. In each case write down the rule that was used to generate the pattern.

a.	…	…	…	12	15	18	…	…	…
b.	…	…	12	24	48	…	…	…	…
c.	…	…	…	…	55	50	45	40	…
d.	…	81	75	…	63	…	…	51	…

4. Create your own patterns using the following rules:
   a. Subtracting the same number each time.
   b. Adding the same number each time.
   c. Multiplying by the same number each time.
   d. Dividing by the same number each time.
   e. First multiplying by a number and then adding another each time.
5. Compare your answers for Activity 3 with one of your colleagues.

Trainer's Note:
This is a crucial observation: you cannot use only two numbers to try and predict the next numbers in a number pattern. Two numbers can result in many different number patterns whereas, in general, three numbers limit the possible number patterns to one only. Be sure that you make this point with the learners.

What have you learned?

In Question 3 you were given at least three of the numbers in the pattern, this is because you will need at least three numbers to establish the rule. Look at the following patterns, if you had only been given the first two numbers you would not have known which of the patterns we wanted to generate.

1 3 5 7 … … …
1 3 9 27 … … …

3. Number patterns that occur in everyday life

There are many everyday situations that give rise to number patterns. Think about Palesa who runs the vetkoek stall at the Bantwana Bami ECD Centre's annual morning market. Palesa's vetkoek sell for R3,00 each. Instead of calculating how much each order costs she has started to make the following table:

Number of vetkoek	1	2	3	4	5	6	7	8	9	10	11
Cost (in rand)	3	6	9	12							

Notice how the numbers in the "Cost" row make a number pattern:

Time needed
15 minutes

Activity 3:
Palesa's vetkoek

Work alone

1. Write down the rule used to generate the number pattern in the cost row above.
2. Complete Palesa's table.
3. Rashida wants to buy 12 vetkoek. Describe at least two different ways in which Palesa can work out the cost.
4. Which of the two ways you described above will help Palesa most if Rashida wants to buy 55 vetkoek?

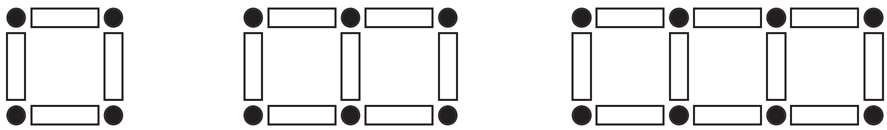
> ***Trainer's Note:***
> *This is another crucial observation. We will revisit this idea many times during the module. It is not important that all learners grasp the subtlety here at this stage, but you should be sure to draw the learners' attention to the two different techniques for completing the table. You will probably find that there will be different people who have used both methods. Use this to help learners to see that there are different ways of "completing a pattern." The approach described in the first bullet is called the **recursive approach** while the approach described in the second bullet is called the **functional/ relational approach**. The purpose for having patterns in the curriculum is to introduce functions and functional relationships. For this reason, it is important that you really stress the functional/relational approach, but do not give it a name at this stage. Also, do not let them think that the recursive approach is not important. It will prove very useful in helping to determine the functional relationships later in the module.*

What have you learned?

In describing two ways in Question 3, you almost certainly found the following:

- You can extend the pattern in the second row of the table by adding three to the numbers in the pattern, or
- You can find the number in the second row by multiplying the number in the first row by 3.

Consider the following shapes that children might make on the carpet using rectangular (white) and circular (black) shaped building blocks. Each shape has a number of open squares in the middle.



Shape 1 (1 square) | Shape 2 (2 squares) | Shape 3 (3 Squares)

We can use a table to record the number of squares in each shape as well as the number of each type of building block needed to make the shape.

Shape number	1	2	3	4	5	6	7	8	9	10	11
Number of squares	1	2	3								
Number of rectangular blocks	4	7	10								
Number of circular blocks	4	6	8								

Time needed 25 minutes

Activity 4:
Building block shapes

Work alone

1. Draw sketches of the next two shapes in the pattern above.
2. Copy and complete the table.
3. Write down, in words, a description of how you completed the table.
4. How many squares will there be in shape number 15? Describe how you know this.
5. How many rectangular blocks will you need to make shape number 15? Describe at least two different ways to work this out.
6. How many circles would be needed to make shape number 20? Describe at least two different ways in which you could work this out.
7. Compare your answers for Question 5 and Question 6 with one of your colleagues.

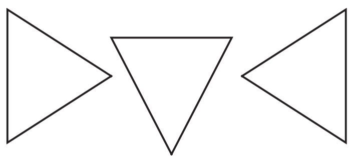
What have you learned?

- There are two kinds of patterns: geometric and numeric patterns. In geometric patterns objects are arranged in order according to a rule. In numeric patterns numbers make up the pattern and there is a rule that relates the numbers.
- Tables of values can help us to identify the rule that relates the numbers in a numeric pattern more easily.
- In the next unit you will look at two ways you have worked with for finding out a particular term. You will think about the advantages and disadvantages or each.

Linking your learning with your ECD work

Making shape patterns

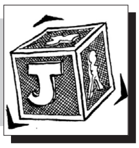
1. Start a pattern using different shape pieces e.g. triangles and squares. Give learners more pieces of the same kind to make more sequences in the same pattern. Let them make as many of their own sequences as they can. When they are finished let them explain how the pattern works e.g. I used one triangle and two squares each time". Encourage them to talk about their different patterns and use their own language to explain the different "rules".
2. Children can use only one shape or object and make a pattern using the same shape in different transformations.



3. Use a collection of four different coloured counters, such as plastic bottle tops that are red, blue, green, yellow. Put each colour into a separate container. Children make a row of four counters using a different colour each time. They think about how many different ways they can do this:

red	blue	green	yellow
blue	green	yellow	red
yellow	red	blue	green
green	yellow	red	blue

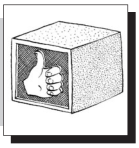
4. With 4 colours and 4 counters in a row there should be 16 possible ways altogether.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about geometric and numeric patterns?
b. Write down one or two questions that you still have about tables of values.
c. How will you use what you learned about geometric and numeric patterns and tables of values in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Distinguish between geometric and numeric patterns				
2. Investigate and analyse both geometric and numeric patterns				
3. Explain and justify patterns observed in both geometric and numeric patterns				
4. Complete tables of values for numeric patterns				
5. Use tables of value for numeric patterns to identify rules used to generate numeric patterns				
6. Answer questions based on information derived from tables of values				

UNIT TWO

Finding rules for number patterns

> ***Trainer's Note:***
> *In this unit we talk again about the different approaches to "finding a rule." There is no "recipe" for determining a rule. You have to see patterns within patterns. We introduce graphs for the patterns we have already worked with to help learners "see" the pattern in a different way and to "look" for the rule. What we would like learners to observe is that the points on the graphs of each of these patterns lie in a straight line. At this stage they may not relate this to the constant change from one term in the pattern to the next; that will come later. Do not try to "make" learners see this at this stage. We'd be delighted if the learners notice that the steeper the line of points the greater the number that is added to get from one term in the number pattern to the next term in the number pattern. We are slowly trying to expose learners to the idea of rate of change and the corresponding gradient of a graph. Do not be too concerned if learners do not see the relationship between the size of the constant and the gradient of the line of points at this stage. This will come with time and there is plenty of opportunity to develop this insight as the module develops.*

In this unit you will address the following:

Unit Standard 7448
SO3:
Devise processes for a general rule. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical and models such as graphs.)

To do this you will:
- draw graphs to describe situations;
- convert between different representations of a numeric pattern including: tables, graphs and verbal descriptions of the rules;
- answer questions about a situation based on the graphs, tables of values and/or verbal descriptions of the situation;
- describe situations using graphs, table and verbal descriptions;
- distinguish between the recursive and the relational approach to finding given terms in a numeric pattern;
- use patterns observed in tables of values and graphs to develop a relational "rule" for generating a numeric pattern.

1. Different ways of finding a particular term in a number pattern

In the module so far you have been asked to think of more than one way to find out a particular term in a number sequence. You will almost certainly have used the following approaches although you may not have used these words to describe them:

a. the recursive approach
b. the relational (or functional) approach.

These two approaches are described below.

Recursive approach

In this approach you continue to use the rule that generates the pattern until you reach the number you are looking for. In the case of Rashida's vetkoek example you could have worked out the price for 15 vetkoek by adding 3 to the previous number until reaching the 15th vetkoek.

Number of vetkoek	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Cost (in rands)	3	6	9	12											
	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3

The recursive approach is easy to use, but it can take quite long - imagine if we used this method to find the cost for 213 vetkoek!

Relational approach
In the relational approach we try to establish a relationship or rule between the position of the term in the pattern (i.e. the term number) and the term itself. You can see this in the case of Rashida's vetkoek:

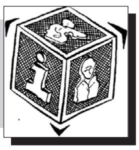
Cost = number of vetkoek multiplied by the cost of one vetkoek

You can write this in a shorter way:

Cost = number of vetkoek × 3

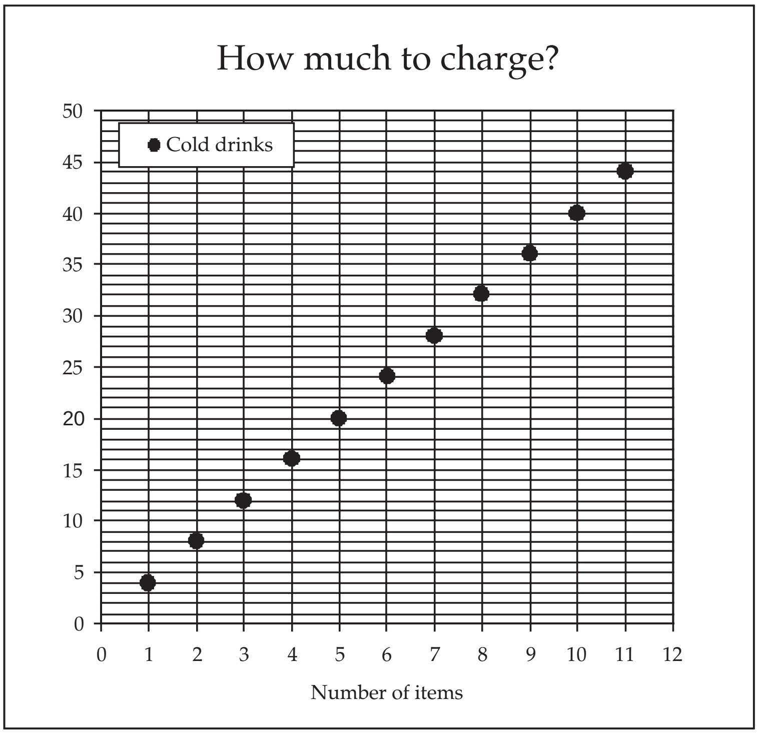
The relational approach has the advantage that you do not have to repeat a large number of simple operations. But it can be difficult to find the relationship or rule.

Our challenge is to study patterns and find the rule or relationship from structure of the pattern. You need to learn to look at the patterns in different ways. Here you will explore how to use tables and graphs to show the structure of patterns.



2. Graphs

Think back to Palesa and her vetkoek in the previous unit. Felicity runs the cold drink stall next door to Palesa. Felicity has also decided to find a quick way to calculate the price of an order but instead of a table she has prepared a graph. This is the graph.



Time needed
25 minutes

Activity 1:
Using graphs to represent number patterns

Work alone

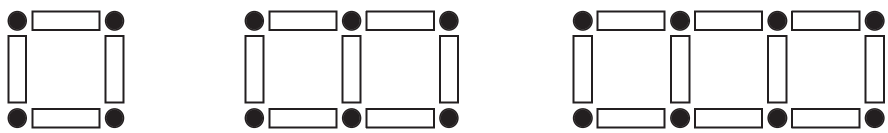
1. Why do you think that Felicity has not joined the points on her graph?
2. Reproduce Felicity's graph on the squared grid paper in your journal. On the same graph add a graph for the costs of Palesa's vetkoek (from unit 1) to this graph.
3. How are the graphs for Palesa's vetkoek and Felicity's cold drinks the same? How are they different?
4. Make a table similar to Palesa's vetkoek table for Felicity's cold drinks.
5. In the notes on the relational approach above you read a rule for finding the cost of any number of Palesa's vetkoek. Find a similar rule for working out the cost of any number of Felicity's cold drinks.
6. Compare your answers with a partner.
7. Make a list of ways in which the tables, the graphs and the rules for Palesa's vetkoek and Felicity's cold drink are the same and different. Look for a connection between these similarities and differences.

Time needed
25 minutes

Activity 2:
Building block shapes again

Work alone

Look at these shapes again:

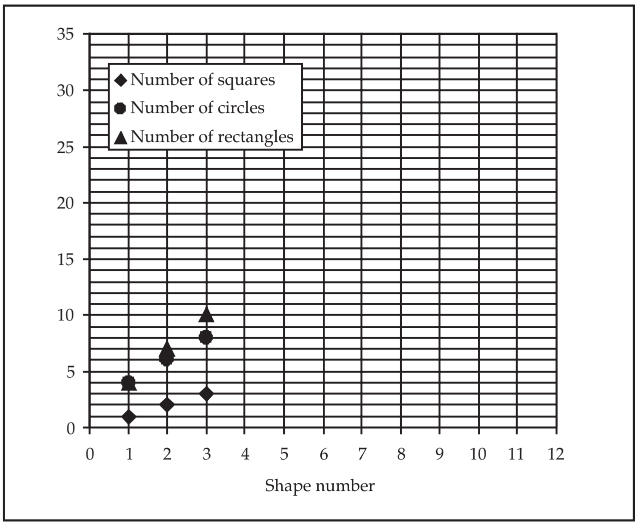


Shape 1 (1 square) | Shape 2 (2 squares) | Shape 3 (3 Squares)

In unit 1 you copied and completed this table. This is what it looks like.

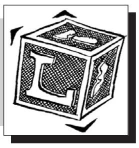
Shape number	1	2	3	4	5	6	7	8	9	10	11
Number of squares	1	2	3								
Number of rectangular blocks	4	7	10								
Number of circular blocks	4	6	8								

1. Draw graphs from the table, for the number of squares, the number of rectangles and the number of circles required for the different shape numbers. The first three points for each pattern have already been marked on this graph for you to see.



2. Find a rule for working out the number of:
   (a) squares
   (b) rectangles
   (c) circles

   for a given shape number.
3. Compare your answers with a partner.
4. Make a list of the ways in which the tables, the graphs and the rules for the three patterns are the same and different. Look for connections between these similarities and differences.



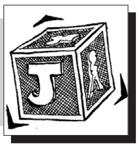
What have you learned?

There are at least two different ways of determining the terms in a numeric pattern:

- The recursive approach relates each element of the pattern to the previous one
- The relational approach relates each element of the pattern to its position in the pattern
- The relational approach has the advantage that you can find any term in the pattern without first having to find all of the terms that come before it. A disadvantage can be that the "relational" relationship is not as easy to establish.

The examples you have studied so far have shown that similar situations will have graphs, tables and rules with similar properties.

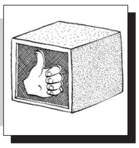
In the next unit you will explore these similarities more carefully.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about graphs?
b. Write down one or two questions that you still have about different representations of numeric patterns.
c. How will you use what you learned about graphs in your every day life and work?



Self-assessment Checklist

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I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Draw graphs to describe situations				
2. Convert between different representations of a numeric pattern including: tables, graphs and verbal descriptions of the rules				
3. Answer questions about a situation based on the graphs, tables of values and/or verbal descriptions of the situation				
4. Describe situations using graphs, table and verbal descriptions				
5. Distinguish between the recursive and the relational approach to finding given terms in a numeric pattern.				

UNIT THREE

More number patterns

Trainer's Note:
The flow diagram later in the unit will help learners "see" the functional relationship between the rule, the table of values, the graph and the "recursive" pattern. However, before introducing the flow diagram we introduce two patterns that are slightly different from the five we have studied so far. The sunflower and telephone call patterns are continuous functions – that is the domain (independent variable/input numbers) can have any real number value (i.e. whole numbers and fractional values). For the five earlier patterns, the values in the domain had to be whole numbers because you can only buy whole Vetkoek. The sunflower is also different from the previous patterns because the rule has a constant in it. Only some of the earlier patterns had constants in them. This is deliberately done to reinforce the notion that some patterns (linear so far) have a constant while others do not.

In this unit you will address the following:

Unit Standard 7448
SO3:
Devise processes for a general rule. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical and models such as graphs.)

SO4:
Represent patterns using different generalized mathematical forms. (graphs, formulae, expressions and other rules for expressing patterns.)

To do this you will:
- draw graphs to describe situations;
- convert between different representations of a numeric pattern including: tables, graphs, flow diagrams and verbal descriptions of the rules;
- answer questions about a situation based on the graphs, tables of values and/or verbal descriptions of the situation;
- describe situations using graphs, tables, flow diagrams and rules;
- identify the common features of tables, graphs, flow diagrams and rules that describe similar situations.

1. Patterns within patterns

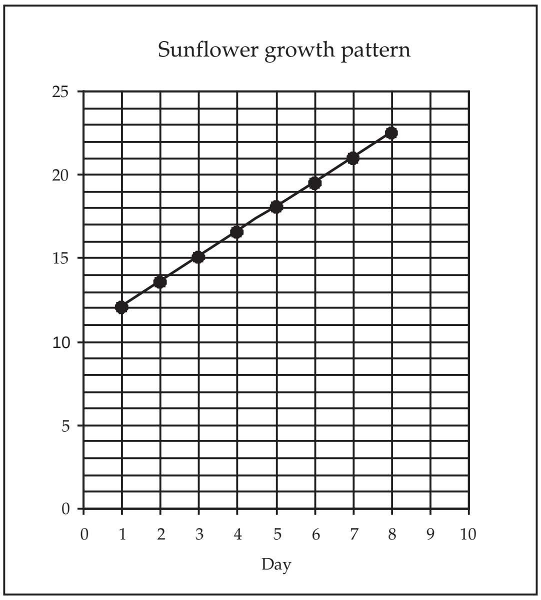
You need to develop the important skill of seeing similarities and differences between different number patterns - patterns within patterns. When you can do this you will be able to
- work out the rules/relationships that generate (make) the patterns and
- work out in your head how the pattern will behave.

In the activities that follow you will explore a number of patterns and in particular relate them back to the patterns already studied in the earlier units.

Time needed
20 minutes

Activity 1:
The sunflower

Work alone
One of the teachers at Bantwana Bami has brought a young sunflower plant to class to show how plants grow. She has put a ruler in the pot. Each day the children mark off how tall the plant is and then the teacher records the height of the plant on a graph. This is the graph for the first 8 days.



You have to use your imagination a bit because plants do not grow as perfectly as the graph suggests.

1. You can see that the teacher has joined the points on her graph. Is this correct? Explain your answer.
2. Make a table of values showing the days and the corresponding height of the plant.
3. Think about Rashida's vetkoek, Felicity's cold drinks and the building block patterns. Which of those patterns is most similar to the sunflower's growth pattern? Explain how the patterns are similar and also how they are different.
4. Work out a rule for finding the height of the plant for any given number of days.

Time needed 30 minutes

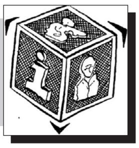
Activity 2: The cost of a phone call

Work alone

Not all of the teachers at the Bantwana Bami ECD Centre own a cell phone and the Centre's telephone may not be used for private calls. Shaheeda has decided to make her phone available for anybody but has decided to charge for the calls. She charges per second and has made the following table to give people an idea of the cost of their call.

Length of call in minutes	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5
Length of the call in seconds	60	90	120	150	180				
Cost of the phone call (in rands)	1,20	1,80	2,40	3,00	3,60				

1. Copy and complete the table.
2. In your journal draw a graph for the cost of a call for a given number of seconds. Did you join the points on your graph? Why?
3. Think about Rashida's vetkoek, Felicity's cold drinks, the building block patterns and the sunflower's growth. Which of those patterns is most similar to the phone call cost pattern? Explain how the patterns are similar and also how they are different.
4. Work out a rule for determining the cost of a phone call for any given number of seconds.
5. Use your rule, or use another way, to find the following:
   (a) The cost of a 480 second phone call
   (b) The cost of a 68 second phone call
   (c) The cost of a 13 minute phone call
   (d) The number of seconds that you can speak for if you have R7,20
   (e) The number of minutes that you can talk for if you have R12,50

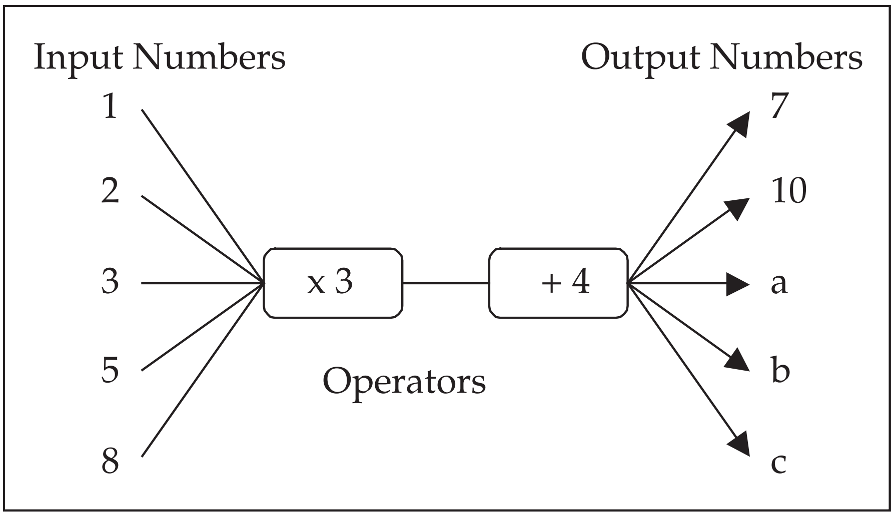


2. Flow diagrams

We sometimes use flow diagrams to summarise a rule. In a flow diagram the input number is your starting number. The operator tells what you have to do to the input number to produce the output number. You can have more than one operator in a flow diagram.

Trainer's Note:
Before going on to the flow diagrams, pause and help learners see the distinction between discreet and continuous functions. You do not need to frighten learners with the terminology. Just draw their attention to the meaning (in context) of these concepts. The flow diagrams are used to help learners "see" the underlying structure in another way. Hopefully you will not have to do too much "showing" of the underlying patterns. But you will almost certainly have to help learners in completing and developing the flow diagrams.

The diagram below shows the flow diagram for the rule: "multiply the input number by 3 and add 4 to the answer"



Time needed
20 minutes

Activity 3:
Flow diagrams

Work alone

1. Work out the values of a, b and c in the flow diagram above.
2. Develop a flow diagram for each of the following patterns (NOTE: only show three input numbers and their corresponding output numbers for each flow diagram)
   a. Rashida's vetkoek
   b. Felicity's cold drinks
   c. The squares in the building block pattern
   d. The rectangles in the building block pattern
   e. The circles in the building block pattern
   f. The height of Helen's sunflower
   g. The cost of a phone call on Shaheeda's phone.
3. Compare your answers for Activity 3 with a partner.

Time needed
20 minutes

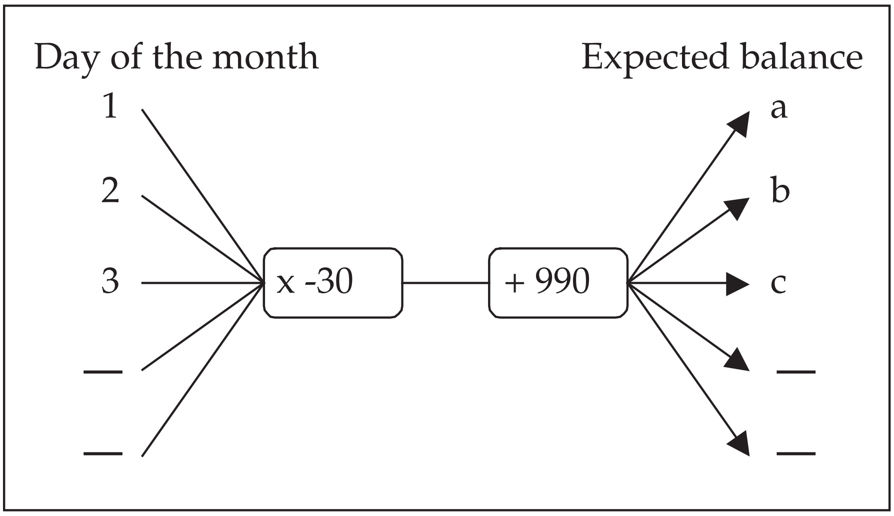
Trainer's Note:
Electricity balance is unusual. It is deliberately unusual! We do not want learners to believe that the values in all number patterns always increase in value. We use the context of topping up the electricity dispenser at the start of the months to watch how the units go down at a constant rate to give a negative gradient. Because the gradient is negative, the first operation in the flow diagram involves multiplying by a negative number. Learners may be uncomfortable with this and may need some scaffolding. Don't make a fuss of the negative number itself. Use the context to help learners see the negative as a directional indicator – i.e. -30 because we have "used up" 30 units each day. How many units do we use in 3 days? The answer is 90 and therefore we must subtract 90 from the amount we started with.

Activity 4:
Electricity balance

Work alone

The Bantwana Bami ECD Centre buys pre-paid electricity units at the start of each month. Each day Shaheeda checks the balance to see if the Centre is using electricity according to the budget. One of the parents has drawn the following flow diagram to help Shaheeda know what the daily balance should be.

Notice in the flow diagram you must first multiply the day of the month by -30 (negative thirty) and then add your answer to 990. The day of the month × -30 corresponds to the amount of electricity used. The 990 is the number of units we started with. It may seem clumsy to do it in this order but that is a limitation of the flow diagram.



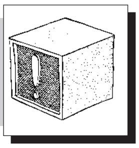
1. Use the flow diagram to calculate the balance on the first three days of the month.
2. What is the balance on the 10^{th} day of the month?
3. On the 17^{th} day of the month the teachers reads the actual balance on the meter and sees that the balance is 510. Has the Centre been using too much electricity or have they been saving? Explain your answer.
4. Compare the electricity flow diagram with the flow diagrams that you developed for the earlier patterns in Activity 3. Which of the earlier patterns is the electricity budget example most like? Explain.
5. Predict what the electricity budget pattern will look like in a table and on a graph.

Time needed 155 minutes

Activity 5:
Comparing answers and predictions

Work in pairs

1. Compare your answers for Activity 4, especially your answers to Questions 4 and 5.
2. Make a table of values for the expected electricity unit balance.
3. In your journal draw a graph for the expected electricity unit balance for each day of the month. Did you join the points on your graph? Why?
4. Were you correct with your predictions regarding the properties of the table and graph? discuss



Stop and Think

Think about your work in this unit and answer the following questions:

1. List the four different ways that we have used to represent a pattern so far.
2. Identify as many "patterns within patterns" that you have observed. We will be summarising these in the next unit and it would be useful for you to think about this before we do so.

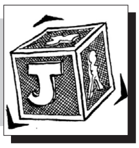
What have you learned?

Flow diagrams are another way of representing the relational relationship in numeric patterns. Flow diagrams "show" the relational rule quite clearly.
There are two different types of situations that we have studied in this module so far:

- Situations that can be described using a flow diagram with a single operator,
- Situations that can be described using a flow diagram with two operators.

The points on the graphs of the patterns that we have studied in this module so far can all be joined by a straight line.

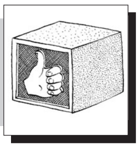
The numbers in the numeric patterns that you have worked with so far can all be created by adding or subtracting the same number to the previous number.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about flow diagrams?
b. Write down one or two questions that you still have about the similarities between tables, graphs, flow diagrams and rules.
c. How will you use what you learned about graphs in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Draw graphs to describe situations				
2. Convert between different representations of a numeric pattern including: tables, graphs, flow diagrams and verbal descriptions of the rules				
3. Answer questions about a situation based on the graphs, tables of values and/or verbal descriptions of the situation				
4. Describe situations using graphs, tables, flow diagrams and rules				
5. Identify the common features of tables, graphs, flow diagrams and rules that describe similar situations.				

UNIT FOUR

Patterns within patterns

Trainer's Note:
Unit 4 summarises what learners will have "seen" in Units 1 to 3. This unit should be completed with great care and patient support of the learners. Work through each of the paragraphs slowly and refer back to Units 1 to 3 after each paragraph to reinforce the remarks. Questions such as: "In which pattern did we see this?" and "Can you think of an example where that happened? Was it the only case?" should help learners a great deal.

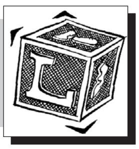
In this unit you will address the following:

Unit Standard 7448
SO3:
Devise processes for a general rule. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical and models such as graphs.)

SO4:
Represent patterns using different generalized mathematical forms. (graphs, formulae, expressions and other rules for expressing patterns.)

To do this you will:
- tell the difference between the recursive and the relational approach to finding given terms in a numeric pattern;
- use patterns that you see in tables of values and graphs to develop a relational "rule" for generating a numeric pattern;
- identify the features of linear patterns including:
  - o The points of the graph of the pattern lying on a straight line
  - o Consecutive elements in the table of values being related by a constant value;
- relate the features of the tables of values, graphs, flow diagrams and rules to each other;
- distinguish between all linear patterns and direct proportion linear patterns;
- distinguish between discrete and continuous relationships.

In the first three units of this module you worked with different kinds of patterns. You explored different ways of showing these patterns and found similarities and differences between the patterns. Let's summarise this before we move on.

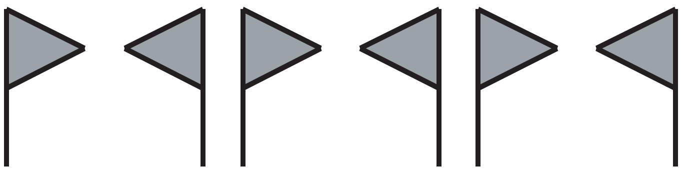


What have you learned?

Geometric and numeric patterns
In geometric patterns, the pattern consists of geometric objects placed in a sequence according to some rule.

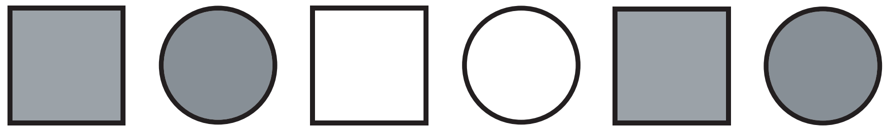
A

B



In pattern A, the rule for generating the pattern is: "rotate the object ( ) clockwise one quarter turn (90˚) each time."

In pattern B the rule for generating the pattern is: "reflect the object ( ) about the vertical line each time."

C


Pattern C consists of a combination of objects – shaded and un-shaded squares and circles. A rule for generating the pattern could be: "repeat the following in sequence: shaded square; shaded circle; un-shaded square; and un-shaded circle."

In numeric patterns, the pattern consists of a sequence of numbers and the rule either relates each number to the number that comes before it or the rule relates the number to its position (term number) in the sequence.

D: 1 3 9 27 81 … … …

The rule for finding number pattern D could be "multiply the previous number by 3" which we can also represent as follows:

D: 1 —[× 3]→ 3 —[× 3]→ 9 —[× 3]→ 27

In number pattern E the rule could be "add 2 to the previous number and divide by 3"

E: 28 10 4 2 … … …

E: 28 —[+ 2]—[÷ 3]→ 10 —[+ 2]—[÷ 3]→ 4

1. Recursive and relational approaches to numeric patterns

In both patterns D and E we find each term in the pattern from the previous term. We call this the recursive approach. The advantage of this approach is that we do not have to think about relating the value of the term to its position in the pattern (term number) – something that could be quite hard especially if your look at pattern E. The recursive approach however would not be very useful if we wanted to know the value of the 25^{th} term as it would take us a long time to find this.

In the relational approach to determining the numbers in a numeric pattern we use a rule to relate the number to its position in the pattern.

F: 5 7 9 11 … … …

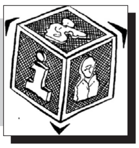
Although we could think of pattern F in a recursive manner:

F: 5 → + 2 → 7 → + 2 → 9 → + 2 → 11

We can also think pattern F in a relational way:

F:	Term number:	1	2	3	4	25
		× 2	× 2	× 2	× 2	× 2
		+ 3	+ 3	+ 3	+ 3	+ 3
	Term:	5	7	9	...	53

While the relational approach may look more complicated at first, and we may have to work harder to find the rule, its advantage in the long run is that it allows us to find, for example, the 25th term without first having to find terms 1 to 24.



2. Using patterns to find the relational rule in numeric patterns

In the many numeric patterns of this unit that you have already explored in this unit you have tried to find a relational rule – i.e. a rule that related the term number to the term in the pattern.

You might have noticed that there have been two kinds of patterns:

Type 1:

Palesa's vetkoek:	3	6	9	12	…
Felicity's cold drinks:	4	8	12	16	…
Number of squares:	1	2	3	4	…
Cost of cell phone call:	1,20	2,40	3,60	4,80	…

Type 2:

Rectangular blocks:	4	7	10	13	…
Circular blocks:	4	6	8	10	…
Helen's sunflower:	12	13,5	15	16,5	…
Crèche's electricity:	960	930	900	870	…

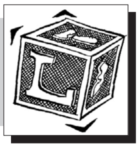
Time needed
20 minutes

Activity 1:
Patterns within patterns

Work in pairs

Answer the following questions about the above patterns:

1. In what way or ways are all of the patterns the same?
2. Look back at your work in Unit 3. In what ways are the graphs of all of the patterns the same?
3. In what way or ways are the Type 1 and Type 2 patterns above different?
4. Look back at your work in Unit 3 again. In what ways are the graphs of the Type 1 and the Type 2 patterns different?



What have you learned?

Linear patterns

You will have noticed at least two things about all of the patterns so far. They are all the same because:

- In all of these patterns you can find the next term in the pattern by either adding a number to the previous term or subtracting a number from the previous term.
- For all of these patterns, the points of their graph lie on a straight line.

We call these linear patterns because the points of these patterns lie on a straight line. This means they are linear functions. In mathematics the word function means relationship.

If you look carefully at the straight lines you will notice that as you move from one point to the next going from left to right, so for every horizontal change of one unit the vertical change is the same – we say the rate of change is constant.

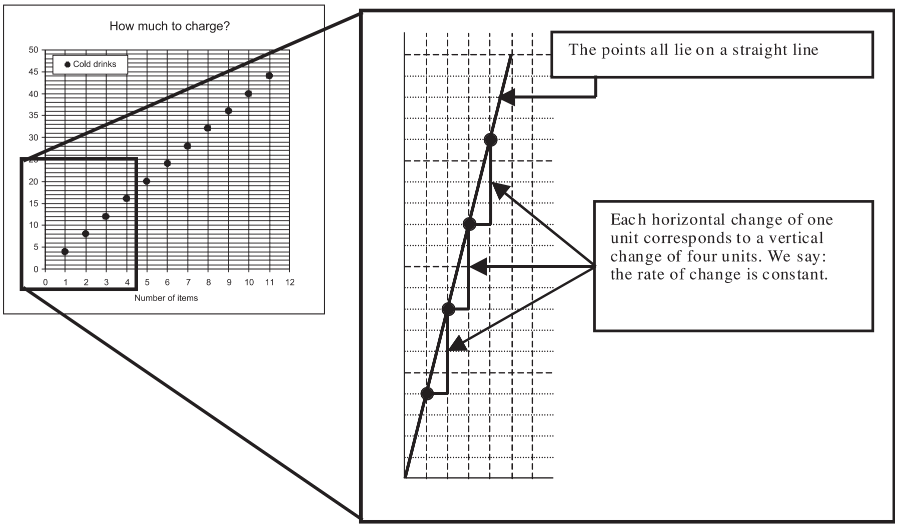
You can also notice the constant rate in the table of values. As the term numbers increase by one in the top row of the table so the values in the bottom row increase (or decrease) by the same amount each time. Look carefully and you will see that the vertical change and the increase in value of the numbers in the bottom row of the table is the same.

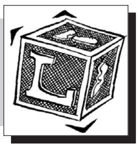
You may want to look ahead to the diagram on page 29 which captures all of these ideas – we will talk about it all again.

Linear patterns/functions have a constant rate of change.
You can see this in the patterns and tables of values because the same amount is added to each term to get to the next one.
You can also see it in the graph of the pattern where the points of the pattern lie on a straight line. The greater the rate of change the steeper the graph is.

Look at the graph below and notice how:

1. The points lie on a straight line.
2. The rate of change is constant.



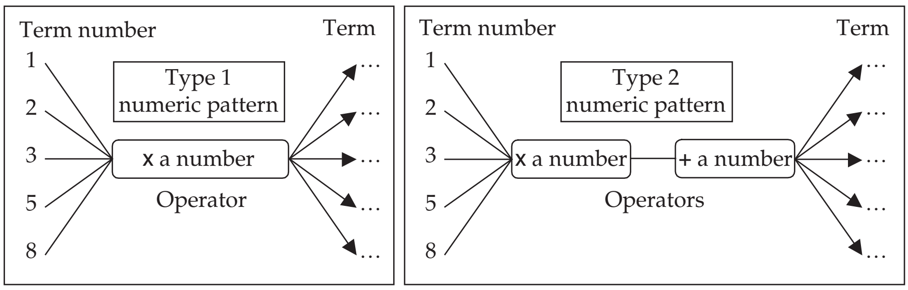


What have you learned?

Two kinds of linear patterns

You will also have noticed that the Type 1 and Type 2 patterns are different. You can see the difference more clearly in their flow diagrams and graphs than in a table.

> ***Trainer's Note:***
> *Use the two diagrams on this page and page 29 to consolidate the underlying patterns that learners have "seen." Think about developing these diagrams on the board using questions like "This constant amount that we added to each term to get to the next term, where do we "see" it on the graph? Where do we "see" it in the table?", "Where do we 'see' it in the flow diagram?" etc.*



In each of the Type 1 numeric patterns you will have noticed that to find the term you only had to multiply the term number by some number. You will also have noticed that the number you multiplied the term number by was the same as the constant rate of change (the number that you had to add to one term to get the next term).

In the Type 2 numeric patterns you will have noticed that to find the term you had to both multiply the term number by some number (the constant rate of change) as well as adding another number.

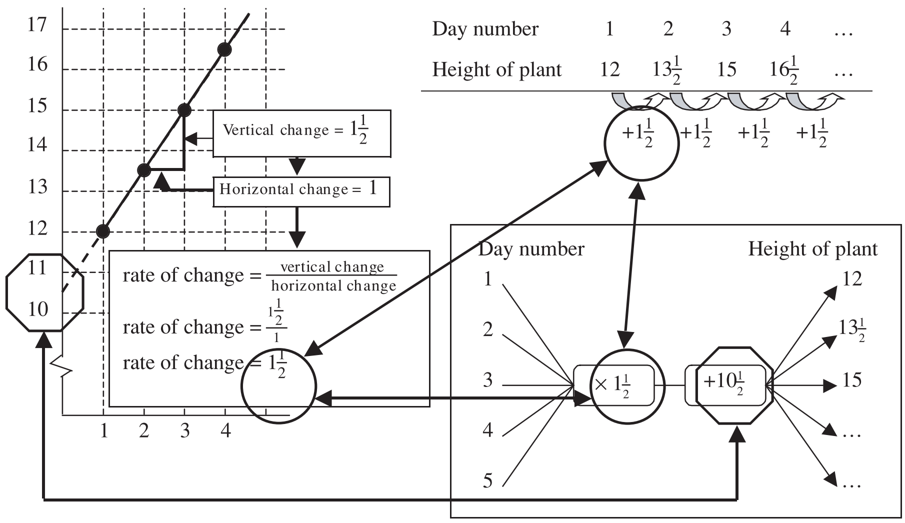
Stop and Think

Think about the graphs of both the Type 1 and Type 2 numeric patterns. In what way are they different?

What have you learned?

Two kinds of linear patterns continued

You have noticed that if you draw and extend a line through the points of the graph of a Type 1 pattern then the line goes through the origin (the point where the horizontal axis and vertical axis meet). If, however, you draw and extend a line through the points of the graph of a Type 2 pattern then the line does not go through the origin. If you look very carefully you will notice that the value at which the line through the points of the graph of a Type 2 pattern meets/cuts the vertical axis is the same as the value we add in the second step of the flow diagram – this value is called the constant. The sketch below shows how the graph, the table and the flow-diagram for the sunflower plant relate to each other.



The Type 1 linear patterns are a special kind of linear relationship called direct proportion relationships. In a direct proportion relationship one quantity increases in proportion to the increase in the other. The rate of change is called the proportionality constant.

3. Equations of linear relationships

There are two equations that describe linear relationships:

All linear relationships can be described by equations of the form:

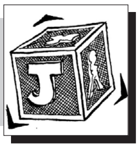
$y = m \times x + c$

where: y is the output number
x is the input number
m corresponds to the rate of change
c is the constant

In some linear relationships there is no constant and the equation becomes:
$y = m \times x$.
We call these linear relationships direct proportion relationships.

Discrete and continuous relationships
There is one last thing that you have noticed as we studied the various numeric patterns in the previous units. With the graphs of some of these patterns we drew a line through the points (joined the points) in the pattern while with others we did not. You have noticed that we did not join the points for Rashida's vetkoek and Felicity's cold drinks with a line, but we did join the points for Helen's sunflower and Shaheeda's telephone costs. This was because you could not buy $1\frac{1}{2}$ or $3\frac{3}{4}$ vetkoek or cold drinks; on the other hand we can measure the height of the plant after $1\frac{1}{2}$ or $3\frac{3}{4}$ days and use the telephone for both $1\frac{1}{2}$ and $3\frac{3}{4}$ minutes.

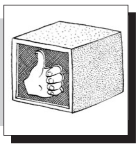
With relationships/functions such as the Rashida's vetkoek and Felicity's cold drinks the input number must be a whole number. We call such relationships/functions discrete relationships/functions. By contrast we call relationships/functions such as the sunflower and the telephone costs continuous relationships/functions because the input number can be any number including fractions.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about the recursive and relational approaches?
b. Write down one or two questions that you still have about linear patterns.
c. What did you learn from this unit about discrete and continuous relationships?
d. How will you use what you learned about linear patterns in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Tell the difference between the recursive and the relational approach to finding given terms in a numeric pattern.				
2. Use patterns that I see in tables of values and graphs to develop a relational "rule" for generating a numeric pattern.				
3. Identify the features of linear patterns including:				
The points of the graph of the pattern lying on a straight line				
Consecutive elements in the table of values being related by a constant value				
4. Relate the features of the tables of values, graphs, flow diagrams and rules to each other.				
5. Distinguish between all linear patterns and direct proportion linear patterns.				
6. Distinguish between discrete and continuous relationships.				

UNIT FIVE

The Crèche Goes on an Excursion – More Linear Functions

Trainer's Note:
This unit draws the learning of the first four units together. The learners work through four different scenarios, each loosely related to an excursion. As the learners work through each activity and reflect on it, they consolidate their understanding of the different characteristics of linear functions:
- *linear functions (all four) and the subset of direct proportion linear functions (petrol consumption);*
- *continuous functions (petrol consumption and recording memories) and discreet functions (zoo visit and ice cream sales);*
- *linear functions with positive gradients (petrol consumption, zoo visit and recording memories)*
- *negative gradient (ice cream sales).*

Some of these scenarios may need some scaffolding as the learners work on them.

In this unit you will address the following:

Unit Standard 7448
SO1:
Recognize, identify and describe patterns in various contexts. (numeric, geometric, patterns from a variety of contexts.)

SO2:
Complete, extend and generate patterns in a variety of contexts. (numeric, geometric, patterns from a variety of contexts.)

SO3:
Devise processes for a general rule. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical and models such as graphs.)

SO4:
Represent patterns using different generalized mathematical forms. (graphs, formulae, expressions and other rules for expressing patterns.)

SO5:
Use general rules to generate patterns. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical models such as graphs.)

To do this you will:
- use the features of the graphs, tables of values, flow diagrams and rules of the functions to recognise and tell the difference between different linear functions:
  - o Linear and direct proportion linear functions;
  - o Discrete and continuous functions;
- find the rules that describe linear functions from the features of the functions evident in their different representations—tables, graphs, flow diagrams and verbal descriptions;
- find output values for given input values and input values for given output values for the different functions in order to answer questions about situations;
- solve contextual problems using various representations;
- understand the role of a constant rate of change in determining different function values.

Time needed
30 minutes

Trainer's Note:
For the first time in this module, we have a table of values that (once completed) does not have the independent variable (distance travelled) in ascending/descending order. This may be a little confusing at first. By drawing a graph from the table of values the learner will clearly see that the relationship is linear. But if the learner first re-arranges the data in the table so that the independent variable was in ascending order, the linear nature of the relationship will still not be clear because the step size in the independent variable is not constant. All of this "confusion" is deliberately introduced so that learners do not fall into the trap of believing that the independent variable in a table of values will always be in order and/or have constant step sizes and will appreciate how one form of representation (in this case the graph) can be more useful than another.

A Bus Trip
Activity 1:
Petrol consumption

Work alone

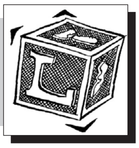
The Bantwana Bami ECD site uses a small bus to transport children and to run other errands. You would like to use the bus for a weekend excursion. In order to budget the expenses for the excursion you need to know how much petrol the bus typically uses for a trip. The bus' petrol tank gets filled every Friday and the amount of petrol used to fill the tank is recorded in the bus log book. The odometer reading is recorded. You can work out the difference between the odometer reading one week and the odometer reading the previous week. This tells you how many km you travelled in that week. A part of the log book is shown here.

Date	Odometer reading (km)	Distance since previous Friday	Petrol added (ℓ)
06-May-05	123815		
13-May-05	124140		39.0
20-May-05	124390		30.0
27-May-05	124515		15.0
03-Jun-05	124915		48.0
10-Jun-05	125003		10.6
17-Jun-05	125183		21.6
24-Jun-05	125433		30.0
01-Jul-05	125793		43.2
08-Jul-05	125958		19.8

1. Copy and complete the table by calculating the number of kilometres travelled during each week.
2. In your journal draw a graph showing distance travelled and the corresponding petrol consumption. You will notice that the points on your graph lie in a straight line.
3. Answer the following questions about the petrol consumption:
   a. Is the relationship between the distance travelled and the petrol consumption linear or not?
   b. Is the relationship between the distance travelled and the petrol consumption continuous or discrete?
   c. Is there a direct proportion relationship between the distance travelled and the petrol consumption or not?

4. Use your answers to question 3 above to do the following:
   a. Develop a flow chart that can be used to calculate the number of litres of petrol needed for any trip with the bus.
   b. Develop an equation that can be used to calculate the amount of petrol needed for any bus trip.

5. If you only have enough money in your budget to pay for 45ℓ of petrol work out how far you can go on a trip.



What have you learned

At first you may not have noticed that the rate of petrol consumption was constant because it was not obvious in the table where the distance "distance travelled since last Friday" varied from one week to the next. This is different from the other tables we have worked with so far – in those tables the numbers are usually written in ascending (and sometime descending) order. But you would have seen from the graph that the rate of petrol consumption was constant.

Because the relationship between distance and petrol used was linear, you could have thought about an equation of the form: $y = m \times x + c$.

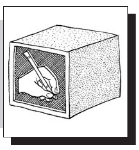
Since a straight line drawn through the points of the graph passes through the origin, the situation was clearly a direct proportion situation and you could have thought about an equation of the form: $y = m \times x$.

You can see that this is an example of a continuous relationship because petrol was used for any distance travelled, including parts of a kilometre.

Time needed
30 minutes

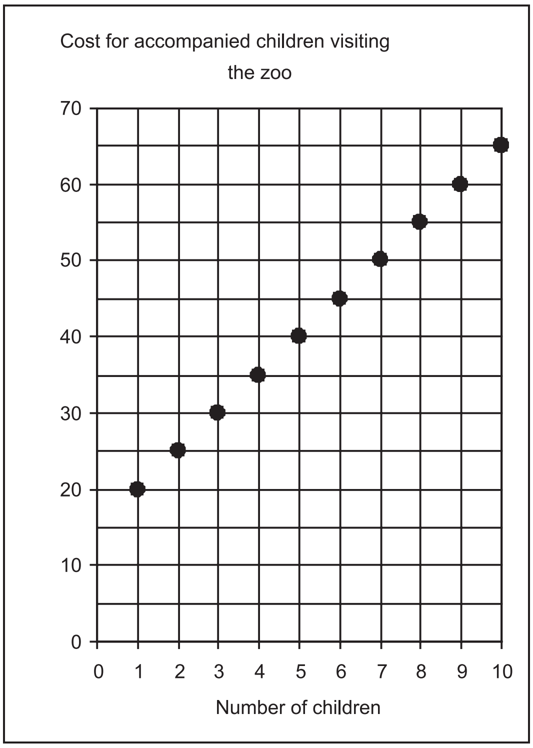
Zoo visit
Activity 2:
Entrance Fees

Write this activity on separate paper and put it in your portfolio.



Work alone

You have decided to visit a zoo on your excursion. The zoo only allows young (pre-school aged) children to visit the zoo if they are accompanied by an adult.



This graph is on the wall of the cashier's office. The cashier uses it to calculate the cost for different groups of children accompanied by an adult.

1. Answer the following questions based on the graph:
   a. Is the relationship between the cost and the number of children in a group linear or not?
   b. Is the relationship between the cost and the number of children in a group continuous or discrete?
   c. Is there a direct proportion relationship between the cost and the number of children in a group or not?

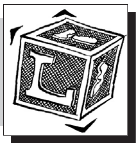
2. Use your answers to question 1 above to do the following:
   a. Develop a flow chart that can be used to calculate the cost for a group with any number of accompanied children.
   b. Develop an equation that can be used to calculate cost for a group with any number of accompanied children.

3. Work out what it costs to visit the zoo if there are 18 children in the group.
4. If you only have R100,00 from your budget to spend on Zoo fees how many children can you take?

> ***Trainer's Note:***
> *The questions in this scenario ask the learners to progress directly from the graph to a flow chart. This may not be very obvious. Of course we hope that they will look at the gradient to determine the coefficient (m) of the variable in the general equation: $y = mx + c$ but you may need to scaffold this. If looking at the gradient does not help the learners then encourage them to develop a table of values from the points on the graph. This should make the process easier. We would like learners to see for themselves that it may help them to draw a table of values, so give them time to think first. We want learners to recognise that different representations will help them to identify certain features more easily, and to feel confident to translate between representations.*

5. Use the flow chart and equation you developed in question 2 above to work out:
   a. The amount that the zoo is charging for the adult who accompanies the children to the zoo.
   b. The amount that the zoo is actually charging for each child (assuming that the children are accompanied)

6. Compare your answers for Activities 1 and 2.



What have you learned?

The rate of increase in costs is constant. This was obvious from the graph since a straight line could be drawn through the points of the graph.

Because the relationship between number of children and cost was linear, you could have anticipated an equation of the form: $y = m \times x + c$.

Seeing that the straight line could be drawn through the points of the graph would not have passed through the origin, c would definitely have had some value. In this case the constant was the price charged for the adult accompanying the children – c = R15,00.

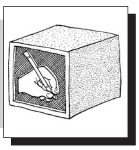
Finally, you realised that this situation was discrete in nature – only whole numbers of children could visit the zoo. This is why the points on the graph have also not been joined.

You can see that this is different from the graph you drew of the petrol consumption.

Time needed
40 minutes

Recording Memories
Activity 3:
Hiring a Video Camera

Write this activity on separate paper and put it in your portfolio.

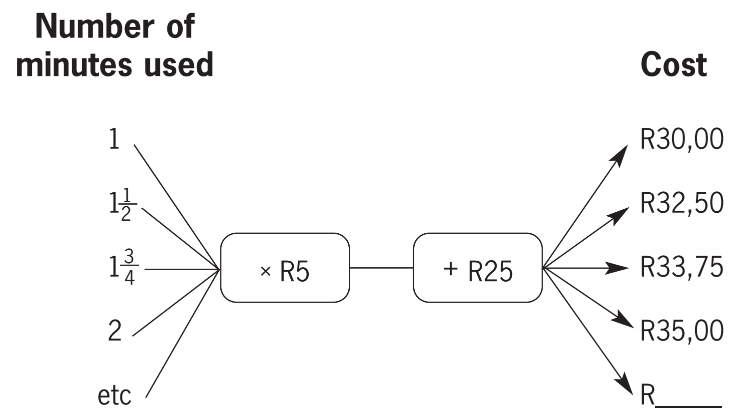


Work alone

You want to record the excursion on video to show to the children. You have to hire a video camera. There are two companies in your neighbourhood that rent these out. These are their advertisements for a day's rental.

HIRE-A-VIDEO CAMERA

Use the flow diagram below to calculate the cost of using our video camera for one day.



REMEMBER-YOUR-DAY

Use the formula below to calculate the cost of using our video camara for one day.

cost = R6,00 × number of minutes used + R12,00

For example:

3 minutes: cost = R6,00 × 3 + R12,00 = R30,00

$3\frac{1}{2}$ minutes: cost = R6,00 × $3\frac{1}{2}$ + R12,00 = R33,00

$3\frac{3}{4}$ minutes: cost = R6,00 × $3\frac{3}{4}$ + R12,00 = R34,50

etc

1. Use the examples in each advertisement to decide which company is the least expensive. Explain how you made your decision.

> ***Trainer's Note:***
> *The only thing that may cause the learners some uncertainty is reading off from the graph the time from which the two companies charge the same amount of money. In the module so far, we have not spent a great deal of time exploring what it means when one graph "lies below" or "lies above" the other, and what it means when the two graphs intersect each other. Scaffolding may be needed to help learners with this.*

2. Answer the following questions based on the flow diagram and equation:
   a. Is the relationship between the cost and the number of minutes of using the video camera linear or not?
   b. Is the relationship between the cost and the number of minutes of using the video camera continuous or discreet?
   c. Is there a direct proportion relationship between the cost and the number of minutes of using the video camera or not?

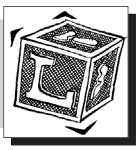
3. Look at the advertisements and your answers to question 2 and discuss what the graphs for each will look like. Talk about how the two graphs will be similar and how they will be different.
4. Copy and complete the table of values below for the two options:

Number of minutes of use	1	$1\frac{1}{2}$	$1\frac{3}{4}$	2	3	$3\frac{1}{2}$	$3\frac{3}{4}$	13	15
HIRE-A-VIDEO CAMERA	30,00	32,50	33,75	35,00					
REMEMBER-YOUR-DAY				30,00	33,00	34,50			

5. In your journal draw a graph for each option on the same set of axes.
6. Use your graph to work out the number of minutes for which both options cost exactly the same amount. Check this on the flow diagram and the formulae in the advertisements.
7. Both companies charge a basic daily fee and a per-minute rate. Copy and complete the table below:

	Basic daily fee	Pre-minute rate
HIRE-A-VIDEO CAMERA		
REMEMBER-YOUR-DAY		

8. Compare your answers for Activity 4 with a partner.



What have you learned?

From the flow chart and the equation used in the two advertisements for renting video machines you will have realised that:

- The relationship for both companies is linear.
- The equation for the remember-your-day company is already in the form: $y = m \times x + c$: (cost = R6,00 × number of minutes used + R12,00).
- The flow chart for the hire-a-video camera company leads to the equation: cost = R5,00 × number of minutes used + R25,00 – written in the form: $y = m \times x + c$.

You have realised that this situation was continuous in nature because any number of minutes, or parts of minutes are possible. Because of this you joined the points of the graph you drew with a straight line.

The basic daily fee is the constant in the equation. For the Hire-a-Video Camera company the constant is R25,00. This is greater than the constant of the Remember-Your-Day Company, which is R12,00. This means you expected the graph of the hire-a-video camera company to start higher up on the vertical axis than that of the remember-your-day company.

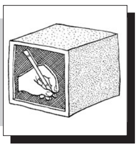
Finally you expected the graph of the hire-a-video camera company to be less steep than the graph of the remember-your-day company. This is because the per-minute rate of the hire-a-video camera company is less than that of the remember-your-day company.

The cost of renting for a short period of time from hire-a-video camera company is greater because the basic daily fee of the hire-a-video camera company is greater. But the per-minute rate of the hire-a-video camera company is less than that of the remember-your-day company. This is why at 13 minutes of use the rental is the same for both companies. At this point the lines from the two graphs meet. After this point the cost from the remember-your-day company is greater.

Time needed
40 minutes

Refreshments
Activity 4:
Working out Ice Cream Sales

Write this activity on separate paper and put it in your portfolio.



Work alone
Moloko sells ice cream at the zoo. He starts each day with a large cold box filled with ice creams. At the end of the day he returns the cold box and remaining ice creams to Veena, his employer. Veena expects Moloko to give her the correct amount of money for the number of ice creams sold.

Trainer's Note:
This scenario may not be very realistic, but the purpose is simply to re-iterate the possibility of a negative gradient. With some support and referring back to the electricity scenario learners should be able to enjoy this one as well.

Veena does not like to empty the cold box each day to count how many ice creams have been sold so she weighs the cold box and ice creams at the start of the day and again at the end of the day. She records this in a register.

During a particular week Veena weighs the box and ice creams at both the start and end of each day and completes her register as follows:

Day of the week	Weight at start of day (kg)	Weight at end of day (kg)	Number of ice creams sold
Monday	6,875	1,625	40
Tuesday	6,875	1,875	40
Wednesday	8,750	1,750	56
Thursday	7,500	2,125	43
Friday	9,375	1,500	63

1. Calculate the weight of a single ice cream. Explain how you find your answer.
2. Calculate the weight of the cold box that Moloko uses to carry the ice creams around in.
3. Use your own understanding and your responses to Questions 1 and 2 to answer the following:
   a. Is the relationship between the number of ice creams in the box and total weight of the box and ice creams linear or not?
   b. Is the relationship between the number of ice creams in the box and total weight of the box and ice creams continuous or discreet?
   c. Is there a direct proportion relationship between number of ice creams in the box and total weight of the box and ice creams or not?

4. On a particular day Moloko starts with 65 ice creams in his cold box. Find the following:
   a. What is the weight of the cold box and the ice creams at the start of the day?
   b. What is the weight of the cold box and the remaining ice creams after Moloko has sold 8 ice creams?
   c. What is the weight of the cold box and the remaining ice creams after Moloko has sold 18 ice creams?
   d. How many ice cream has Moloko sold if the weight of the cold box and the remaining ice creams is 4,125kg?

5. Develop equations that Moloko can use to predict the weight of the cold box and the remaining ice creams after selling a number of ice creams if:
   a. he starts the day with 65 ice creams
   b. he starts the day with 45 ice creams
   c. he starts the day with 38 ice creams.

6. Draw a graph for each of the three situations described in Question 5.
7. Discuss how the values of m and c in the equation: $y = m \times x + c$ effect the graph of the relationship.
8. Discuss how the values of m and c in the equation: $y = m \times x + c$ effect a table of values for the relationship.

What have you learned?

The situation of Moloko's ice cream is interesting. We can think about it in two ways. If we think about the number of ice creams in the box and the corresponding weight of the box then it is quite obviously linear. The weight of the box is constant and the total weight is found by adding the weight of the box to the number of ice creams in the box multiplied by the weight of one ice cream. So the equation for this is

total weight = weight of one ice cream × number of ice cream + weight of the empty box

i.e. $y = m \times x + c$

with: y = total weight,
x = number of ice creams in the box
m = weight of one ice cream
c = weight of the empty box

In question 4, however, you developed equations that can be used to find the weight of the box after a given number of ice creams have been sold if you know how many ice creams there were to begin with:

remaining weight = initial weight – weight of one ice cream × number of ice creams sold

or:

remaining weight = – weight of one ice cream × number of ice creams sold + initial weight

Although this second form of the equation is less attractive (starting with a negative sign), by writing it like this we can see the general pattern more clearly.

i.e. $y = - m \times x + c$

You noticed that the graph sloped down from left to right. This is different to other graphs you have drawn up until now.. You may have realised that the reason that the graph slopes differently is because of the "–" sign in the equation.

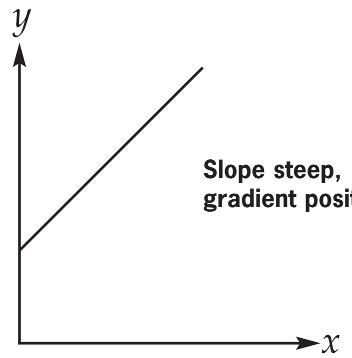
In question 5, we also changed the initial weight of the box with ice creams by changing the original number of ice creams in the box. So, you drew three graphs which all had the same slope but which all started at a different place on the vertical axis.

In Unit 4 we summarised the properties of the different relationships studied so far in this module.

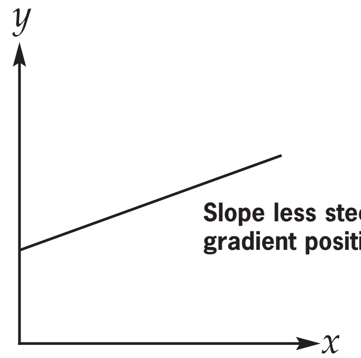
In this unit we have looked at a number of situations and have been able to interpret them more easily because we thought about the nature of the relationship.

Look at how these graphs sum up why some graphs slope differently.

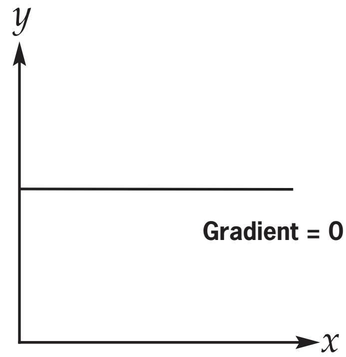
When y changes rapidly with respect to x the slope is steep, and the gradient is positive.



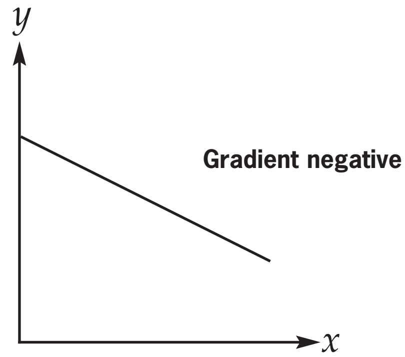
When y changes slowly with respect to x the slope is less steep, but the gradient is still positive.

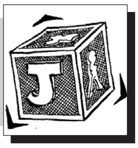


When there is no change in y as x changes, the slope is flat and the gradient is zero.



When y decreases as x increases, the gradient is negative.

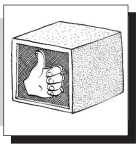




Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about different linear functions?
b. What did you learn from this unit about representing linear functions?
c. Write down one or two questions that you still have about output and input values.
d. Write down one or two questions that you still have about constant rate of change and linear functions.



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Use the features of the graphs, tables of values, flow diagrams and rules of the functions to recognise and tell the difference between different linear functions:				
Linear and direct proportion linear functions				
Discrete and continuous functions				
2. Find the rules that describe linear functions from the features of the functions evident in their different representations – tables, graphs, flow diagrams and verbal descriptions.				
3. Find output values for given input values and input values for given output values for the different functions in order to answer questions about situations.				
4. Solve contextual problems using various representations.				
5. Understand the role of a constant rate of change in determining different function values.				

UNIT SIX

Some Non-linear Functions

In this unit you will address the following:

> ***Trainer's Note:***
> *This short unit serves only one purpose. It is intended to prevent the learner falling into the trap of thinking that all relationships are linear. We introduce scenarios that lead to non-linear graphs to show that all of the "patterns within patterns" observed in the module so far apply only in the special case where relationships are linear. It is important not to spend too long on this unit as we are not going to study any of the other relationships in any detail now.*

Unit Standard 7448

SO1:
Recognize, identify and describe patterns in various contexts. (numeric, geometric, patterns from a variety of contexts.)

SO2:
Complete, extend and generate patterns in a variety of contexts. (numeric, geometric, patterns from a variety of contexts.)

SO3:
Devise processes for a general rule. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical and models such as graphs.)

SO4:
Represent patterns using different generalized mathematical forms. (graphs, formulae, expressions and other rules for expressing patterns.)

SO5:
Use general rules to generate patterns. (Processes include: systematic counting, sequencing numbers, tables, drawings, pictures, classification, organized lists, mathematical models such as graphs.)

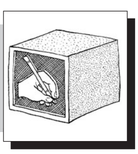
To do this you will:
- recognise and tell the difference between non-linear and linear functions;
- recognise and tell the difference between the graphs and equations of exponential, quadratic and inverse proportion functions.

1. Non-linear functions

So far you have only studied linear functions. In this unit you will look at real life examples of relationships that are not linear. You will see that the graphs, tables of values and equations of these non-linear relationships are different from the graphs, tables of values and equations of the linear examples of relationships.

Time needed 30 minutes

Activity 1:
Comparing Salary Options



Write this activity on separate paper and put it in your portfolio.

Imagine that you have been offered a job at two different ECD centres. The first centre, Sunny Days, offers you a fixed salary of R2 000.00 per month. The second centre, Rainbow House offers to pay you R20.00 in the first month, double that in the second month (i.e. R40.00) and so on, doubling your salary every month.

1. Discuss with a partner which salary option you think is best and why.

Let us investigate these two options more carefully:

2. Copy and complete the following table:

Sunny Days centre						
Month	1	2	3	4	5	6
Salary for the month	2 000,00	2 000,00	2 000,00	2 000,00	2 000,00	
Total salary earned at end of period	2 000,00	4 000,00	6 000,00			

3. Work out the following for the Sunny Days centre:
   a. The amount you will earn in the 12th month.
   b. An equation that you can use to find the total salary earned after n months. (Notice that this is a linear relationship.)
   c. The total salary earned after (i) 8 months and (ii) 12 months.

4. Copy and complete the following table:

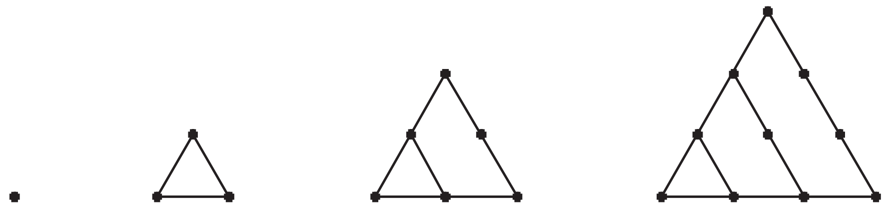
Rainbow House centre						
Month	1	2	3	4	5	6
Salary for the month	20,00	40,00	80,00	100,00	200,00	
Total salary earned at end of period	20,00	60,00	140,00			

5. Find the following for the Rainbow House centre:
   a. The amount you will earn in (i) the 8th month, and (ii) the 12^{th} month.
   b. The total salary earned after (i) 8 months and (ii) 12 months.

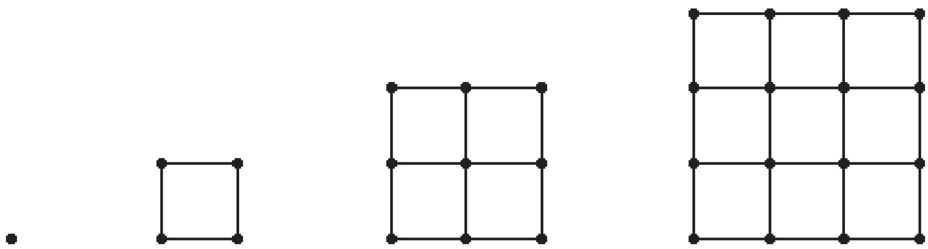
6. In your journal draw a graph for each centre comparing the total salary earned with the number of months worked.
7. Compare the graphs and the tables of values for the Sunny Days and Rainbow House centres. In what ways are they different?
8. Read from your graph the number of months after which you will have earned approximately the same total salary.

2. More Geometric Patterns – Triangular and Square numbers

"Triangular numbers" are numbers that correspond to the number of dots needed to make the triangles in the geometric patterns below:



"Square numbers" are numbers that correspond to the number of dots needed to make the squares in the geometric patterns below:



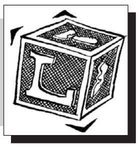
Time needed 30 minutes

Activity 2:
More Geometric Patterns – Triangular and Square numbers

1. Draw the next two shapes in each of the above geometric patterns.
2. Complete the table below:

Number of dots in the side of the shape	1	2	3	4	5	6	7	8	9
Triangular numbers (i.e. total number of dots in triangles)	1	3	6						
Square numbers (i.e. total number of dots in squares)	1	4	9						

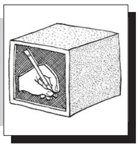
3. Describe carefully how you find the values for the shapes with 7, 8 and 9 dots in the side of the shape.
4. In your journal draw a graph showing the relationship between the number of dots in the side of the shape and the total number of dots in the shape for both the triangular numbers and the square numbers.
5. Discuss how the tables of values and the graphs of these relationships are different to the tables of values and the graphs of the linear relationships.
6. Compare your answers to the questions in Activities 1 and 2 with a partner.



What have you learned?

You can see that the number patterns made by both the triangular and square shapes are clearly not linear because they are not straight lines on a graph. Now let's look at a real life example of this.

Time needed 30 minutes



Activity 3: Sharing your winnings

Write this activity on separate paper and put it in your portfolio.

Bantwana Bami ECD site has decided to hold a "guess the score" competition for the final match in the soccer championships. Each person who buys a ticket must predict the final score. The person or people who guess the final score correctly will either win all of the prize money or share it evenly among all those who guessed the correct score.

Because this is a fundraising event the prize money is not very large - in fact it is only R16.00! In this example the prize money is small. In real life, the relationships are the same but the numbers are much larger.

1. How different will the amount be if you are the only person to win or if two people win? Express your answer as a fraction of the total prize money.
2. What fraction of the total prize money will you win if you and two other people (i.e. three in total) all guess the correct score?
3. What fraction of the total prize money will you win if you and three other people (i.e. four in total) all guess the correct score?
4. What happens to the fraction of the prize money as the number of people who win increases?
5. In your journal draw a graph to show how the share of the R1600 prize money changes as the number of people who win increases.
6. Compare your answers to the questions in Activity 4 with a partner.

Reflecting on the relationships you have seen in this unit

In this unit you have worked with three different families of non-linear functions.

The table below shows the general equation for each of these families and specific equations that correspond to the examples in the activities in this unit.
Remember in Module 1 you worked with powers of numbers, such as 102. You know that this means 10 × 10. To do the next activity think about the following:

x^2 means $x \times x$ … i.e. $x^2 = x \times x$
$x^3 = x \times x \times x$ and
$x^4 = x \times x \times x \times x$ etc.

Time needed
30 minutes

Activity 4:
Reflecting on the different relationships in this unit.

Work with a partner:

1. Copy and complete the table:
   a. Work out the output values of each of the specific equations for the given input values.
   b. Match each of the examples in the activities in this unit to its equation.

Function type and equation	Specific equation	Input number 1	2	3	4	5	6	Examples in the activities
Exponential $y = a \times b^x$	$y = 10 \times 2^x$							
	$y = \frac{20}{3} \times 3^x$							
Quadratic $y = ax^2 + bx + c$	$y = x^2$							
	$y = \frac{1}{2}(x^2 + x)$							
Inverse proportion $y = \frac{k}{x}$	$y = \frac{16}{x}$							

What have you learned?

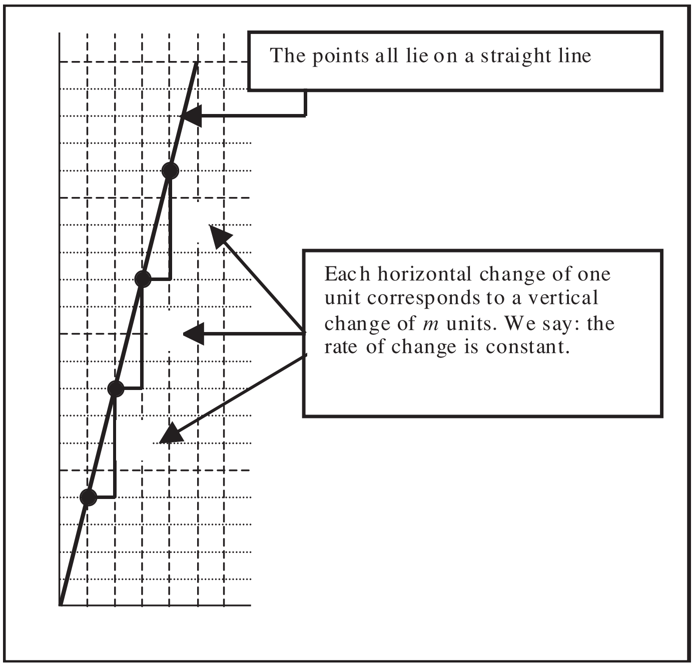
In this unit you have learned that some functions are non-linear. You have seen that the equations, the tables of values and the graphs of these functions are different to the linear functions as a result you can also see more clearly that linear functions have a constant rate of change. Which we can see in a table of values when we notice the following pattern:

+1 +1 +1 +1 +1 +1 +1

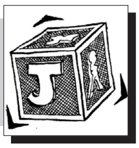
Input number 1 2 3 4 5 6 7 8 9 10 11 12 15
Output number

+m +m +m +m +m +m +m

You know that linear functions have a constant rate of change and that is why the points of the graphs of the functions lie on a straight line.



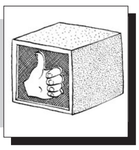
Non-linear functions do not have constant rates of change and therefore the points of their graphs do not lie on a straight line but rather on a curve with a different shape.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about the difference between non-linear and linear functions?
b. Write down one or two questions that you still have about the difference between the graphs and equations of exponential, quadratic and inverse proportion functions



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Recognise and tell the difference between non-linear and linear functions				
2. Recognise and tell the difference between the graphs and equations of exponential, quadratic and inverse proportion functions				

UNIT SEVEN

Algebraic Techniques

Trainer's Note:
This unit is intended to introduce learners to the most elementary of algebra techniques without losing an intuitive approach to algebra. You will notice that we do not teach a "method" or "rule" for solving a linear equation. We rather want the learners to think about what they are doing. We want them to realise that solving a linear equation is really no more than "undoing" the steps. Toward the end of the unit we do, however, teach a rule for factorising the difference of squares because there is no intuitive way of thinking about this. We introduce factorisation to give learners an opportunity to manipulate expressions and see if they are equivalent. This skill is listed in the Unit Standard range statement.
Throughout the unit we want the learners to see that they can answer all of the questions about equivalence substitution and by trial-and-improvement techniques, but that algebraic techniques can help to make the task less tedious. We expect learners to realise that there are more elegant and efficient ways used by mathematicians to manage these kinds of problem. So, this unit is more like a window into the world of a mathematician than a set of vital skills for the ECD practitioner.

In this unit you will address the following:

Unit Standard 7453
SO1:
Form and use algebraic equations and inequalities to represent and solve problems. (simple linear equations and inequalities.)

SO2:
Manipulate algebraic expressions to find equivalent forms. (common factors, products and grouping using associative, distributive and commutative properties.)

SO3:
Select and use algebraic formulae to solve problems. (Substitution into any formula, solve for one variable, supplied formulae from any context.)

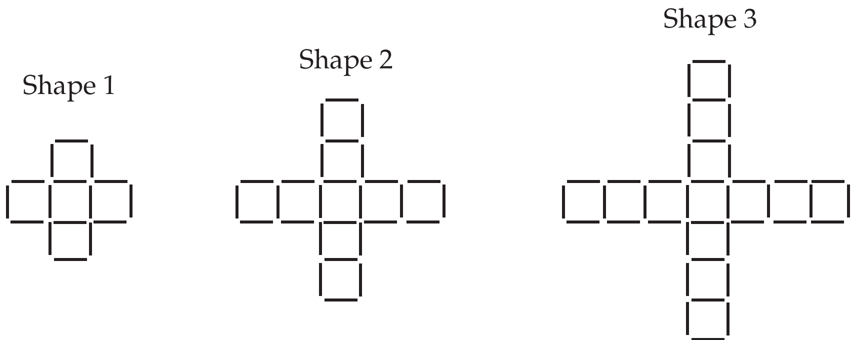
To do this you will:
- identifying the difference between like and unlike terms;
- gathering and adding/subtracting like terms;
- using the meaning of exponents;
- simplifying terms that involve both multiplication and division with exponents;

and
- solve equations using the "trial and improvement" technique;
- solve equations using the "isolating the unknown" technique (together with flow diagrams);
- solve simple inequalities using tables of values;
- factorise expressions in order to show equivalence of expressions using both the "common factor" and "difference of squares" techniques.

1. Different but equivalent rules

Activity 1:
Exploring geometric patterns

Ishmael has made the following sequence of patterns using pick-up sticks on the mat.



1. Copy and complete the table below.

Shape number	1	2	3	4	5	6
Number of sticks	16	28				

2. Develop a rule for finding out the number of sticks for any given shape number.

What have you learned?

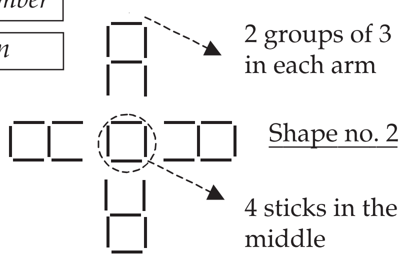
Thandiswa and Richard have each developed a different rule for the pattern in the table above:

Thandiswa's rule was $4 + 4 \times 3 \times \textit{shape number}$

which she has summarised as . $4 + 12n$

She explained her rule like this:

'I saw 4 sticks in the middle and then counted groups of 3 sticks on each arm. The number of groups of 3 was the same as the shape number. I also noticed that there were 4 arms in each shape.

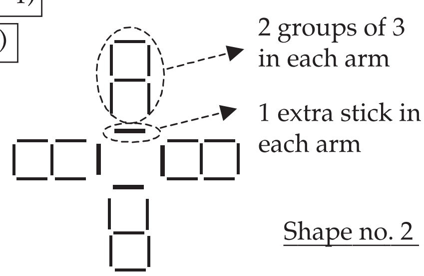


Richard's rule was $4 \times (3 \times \textit{shape number} + 1)$

which he has summarised as . $4 \times (3n + 1)$

He explained his rule as follows:

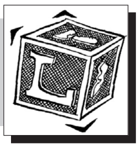
I saw 4 arms. Each arm has groups of 3 sticks. The number of groups of 3 was the same as the shape number. There is 1 extra stick at the end of the row.



Activity 2: Checking the rules

1. Complete the table below to check Thandiswa and Richard's rules give the correct number of sticks.

Shape number	1	2	3	4	5	6
Number of sticks	16	28				
Thandiswa's rule						
Richard's rule						



What have you learned?

You completed the table and realised that both of the rules give the correct number of sticks. This means that each of these rules is the same.

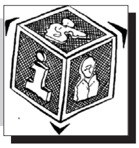
Because the rules are the same, we can write:

In the activity above you used the table of values to show that the rules are the same.

There is another way of showing that the rules are equivalent – we will spend the remainder of this unit developing the techniques to do so. The approach is called the algebraic approach.

Mathematicians prefer to use algebra to show that these rules are equivalent (the same). You can use algebra to change the rules so that they look exactly the same. In this unit you will develop the algebraic techniques (methods) that you need to show that the rules are equivalent algebraically.

In this unit you will learn a lot of new vocabulary. You may want to keep a list of all of the new words and their meaning in your journal, or in the margins of this manual.



2. Terms

All algebraic expressions are made up of quantities which are called terms. They are linked to each other by plus and minus signs. A plus or minus sign is called the operator. For example:

$2x + 1 - y$ has 3 terms

$\frac{3x}{2} - x^2$ has 2 terms

$3 \times 2a + 6b$ also has 2 terms.

Time needed 15 minutes

Activity 3: Counting terms

1. Give the number of terms for each of the following expressions:

(a) $3x^2 + 2x - 1$

(b) $5xy - 2$

(c) $3 \times 2a$

(d) $6a - 7b + \frac{1}{2}a + 3b - 5$

(e) $4a - 3a^2 + 2b - b^2$

(f) $2 \times a \times b - 4 \times a^2$

(g) $6x^2y^2 - 3x + x^3 - 4x \times y$

(h) $p + q + 2p - 3pq + \frac{1}{2}p - 4 \times 2q$

(i) $y + 2y^2 - \frac{3}{y}$

(j) $\frac{3x}{2y} + 1$

2. Check with a partner if you got the same answers.

3. Like and unlike terms

Terms which have the same letter(s) are called like terms. For example:

$2x$ and $+ 3x$ are like terms in the expression: $2x + 3x$
12 and +4 are like terms in the expression: $12 - 2a + 4$

Terms which do not have the same letter(s) are called unlike terms. For example:

$2x$ and $+3y$ are unlike terms in the expression: $2x + 3y$
$-5x$ and -10 are unlike terms in the expression: $-5x - 10$

Some expressions can have a mixture of like and unlike terms. For example:

In the expression: $2x - 3 + 4x$, the terms $2x$ and $+4x$ are like terms, but –3 is unlike.

In the expression: $\frac{1}{2}a + 2b + 6a - b$, the terms $\frac{1}{2}a$ and $+6a$ are like, and the terms $2b$ and $- b$ are also like, but $\frac{1}{2}a$ and $2b$ are unlike.

Time needed 15 minutes

Activity 4: Identifying like and unlike terms

1. State whether the terms in the expression below are like or unlike:

(a) $2a + 5a$

(b) $2a + 5$

(c) $3x - y$

(d) $3a - b$

(e) $6m - 2m$

(f) $2a^2 + 4a$

(g) $10 + 2$

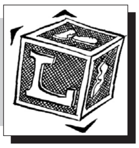
(h) $5xy + 4x$

(i) $x^2 + 3x^2$

(j) $3ab - 3a + 2b$

2. In each of the expressions below, two of the terms are like. Identify the like terms.

(a) $3x + 5xy - x$	(b) $2ab + a^2 - 3b + 6a^2$
(c) $7a + 3b + 5 - 3a$	(d) $2x - 3y - 2z + 4xy + 9y$
(e) $2n^2 - 6m + 3 + 10m$	(f) $a^2 + 4a + 3a + 2$
(g) $10 + 2$	(h) $5xy + 4x$
(i) $x + 8 + y - 2$	(j) $-ab + 6b - 2a^2b + 5a + 10a^2b$



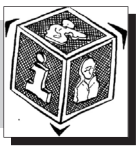
What have you learned?

In Activity 3 you counted three terms in number a. and four terms in number e. Number h. was the longest with seven terms.

You also noticed that you always write down a term with the operator – i.e. the plus or the minus sign before it. If there is no operator written in front of the first term you know that it is a plus (addition) sign. You can leave it out. That is why you can write $3x$ instead of $+3x$. That is also why you can talk about the "terms $3x$ and $+3x$ as like terms" .

A convention in algebra is a habit. It is something that people do that is always the same. So when you leave out the plus sign in $3x$ that is a convention.

Algebraic convention:
In algebra the operator before the first term in an expression is assumed to be "plus" if no operator is shown:
$2a + 3b$ = "+" $2a + b$



4. Simplifying expressions

You can simplify expressions by adding or subtracting the like terms. Only like terms can be added or subtracted in order to get a single term. Unlike terms cannot be added or subtracted. For example:

Like terms: If I have three a's ($3a$) and add another two a's ($2a$) I will have five a's ($5a$). ($3a + 2a = 5a$)

Unlike terms: If I have three a's ($3a$) and add another two b's ($2b$) I will have three a's ($3a$) and two b's ($2b$) written: $3a + 2b$. This expression cannot be simplified any further.

Here is another convention.

Algebraic convention:
In algebra the expression $4a$ is a shorthand for $a + a + a + a$ or $4 \times a$.

Read through these examples of simplifying expressions.

$4x + 2x$ can be simplified into one term: $6x$. You write $4x + 2x = 6x$.
$4a$ and $+2a$ in the expression $4a + 3b + 2a$, can be simplified into one term: $6a$. You write $4a + 3b + 2a = 6a + 3b$. This $6a + 3b$ cannot be simplified further because $6a$ and $+3b$ are unlike terms.

$4p + 2q - p + 3q = 3p + 5q$ because $4p$ and $-p$ are like terms. You can see that $-p$ is short for $-1p$. Also $+2q$ and $+3q$ are like terms. In the simplified expression $3p$ and $+5q$ are unlike terms. This means you cannot simplify the expression any further than $3p + 5q$

Here is another convention

Algebraic convention:
It is convention in algebra to write terms in alphabetical order and to write terms with no letters right at the end. For example we write: $2a + 3b - 10$. The term $2a$ come before $3b$ and the number term (-10) comes right at the end.

Time needed 20 minutes

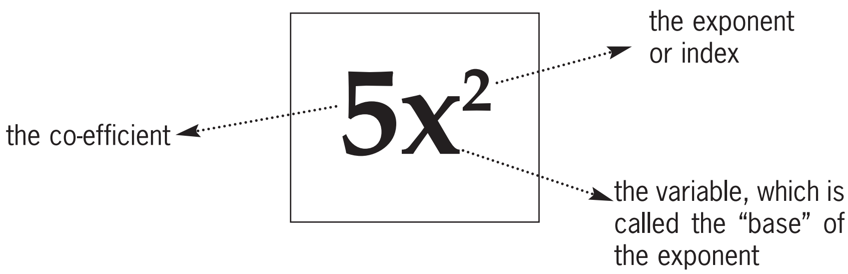
Activity 5: Simplifying expressions

1. Simplify the following by adding or subtracting the like terms.

(a) $x + x$
(b) $5y + y$
(c) $3u - 2u$
(d) $4p - 3p$
(e) $x + x - x$
(f) $2x - 2x$
(g) $5x - x + 6$
(h) $y + 2 + y$
(i) $2t + 1 - t$
(j) $x + y + x$
(k) $x + y - x$
(l) $x + y + x - y$
(m) $x + 1 - x$
(n) $1 + y + y$
(o) $3z - z + 3z$
(p) $t - t + t$
(q) $9d - 7d - 2d$
(r) $5 + 5x - 5 - 4x$
(s) $y + y + z + z$

5. Introduction to Exponents

An exponent (or index) is a little number which is placed on the upper right of a variable (letter) to show repeated multiplication. The big number, in front of the variable is not an exponent. It is called the co-efficient.



The "little number" ^{2} in the term x^2 shows there are two x's which are multiplied together i.e. $x^2 = x \times x$.
The "little number"^{5} in the term $3a^5$ shows there are five a's which are multiplied together i.e. $3a^5 = 3 \times a \times a \times a \times a \times a$.

There is no "little number" in the term x, but you can write x as x^1 because there is only one x which is multiplied. You do not have to write the exponent if it is a 1. Instead of writing y^1 you simply write y.

Here is another convention.

Algebraic convention:
When the exponent is a 1 instead of writing y^1 we simply write y

Multiplying terms with exponents:
Sometimes you need to simplify an expression that has a term like: $x^2 \times x^3$. Remember that: x^2 means $x \times x$ and x^3 means $x \times x \times x$. So $x^2 \times x^3$ actually means $x \times x \quad \times \quad x \times x \times x$. You can simplify this to x^5 because
$x \times x \quad \times \quad x \times x \times x = x \times x \times x \times x \times x = x^5$.

Dividing terms with exponents:
You may need to simplify an expression that has a term like: $a^6 \div a^2$.

Remember that a^6 means $a \times a \times a \times a \times a \times a$ and a^2 means $a \times a$, so $a^6 \div a^2$ actually means $\frac{a^6}{a^2}$ which you can write as: $\frac{a \times a \times a \times a \times a \times a}{a \times a}$.

In the same way that $\frac{3}{3} = 1$, it is also true that $\frac{a}{a} = 1$.

So if $\frac{a}{a} = 1$ it is logical that

$$\frac{a \times a \times a \times a \times a \times a}{a \times a} = \frac{a}{a} \times \frac{a}{a} \times a \times a \times a \times = 1 \times 1 \times a \times a \times a \times a = a^4$$

In the expression $x^2 \times y^3$ x and y are called the bases. You can only simplify in this way if the variables are the same. So, be careful. If you see an expression with terms like: $x^2 \times y^3$ or $a^6 \div a^2$ you cannot simplify these because the bases x and y are different and the bases a and b are also different.

Time needed
15 minutes

Activity 6:
Multiplying with exponents

1. Simplify each of the following terms as shown above:

(a) $x^3 \times x^4$ | (b) $a^5 \times a^3$
(c) $m^2 \times m^3$ | (d) $b \times b^4 \times b^3$
(e) $x^7 \div x^3$ | (f) $y^5 \div y^3$
(g) $p^4 \div p^3$ | (h) $a^7 \div a^3$

2. Did you notice any pattern in question 1? If so, describe what you noticed.

What have you learned?

The rule for multiplying with exponents

You noticed in Activity 6 above, that when you multiply expressions with like bases, you added their exponents. You can summarise this using the rule: $x^a \times x^b = x^{a+b}$

The rule for dividing with exponents

You also noticed in activity 6 above that when you divide expressions with like bases, you subtract their exponents. You can summarise this using the rule: $x^a \surd x^b = x^{a-b}$

6. Solving equations

You remember at the beginning of this unit Thandiswa developed the rule $12n + 4$ to find the number of sticks used in any shape in the pattern.

To find the number of sticks in the 8^{th} shape, she can substitute 8 for n in her rule as follows:
Number of sticks in the 8th shape = $12 \times 8 + 4$ which is equal to 100.

Now Thandiswa also wants find out which number shape in the pattern she can make using exactly 124 sticks.

This time, she needs to find a value for n so that: $12n + 4 = 124$.

To find her answer we say that she must "solve the equation" $12n + 4 = 124$.

This means that she must follow the process to find the value of n that will make the statement true. We say that a value of the variable that solves the equation "satisfies" the equation.

Solving equations by trial and improvement

One method that you can use to solve equations is called trial and improvement. In this method you "guess" a solution and then check to see if it satisfies the equation. For example:

Solve: $5x + 12 = 62$

Guess 1: Guess a number $x = 3$, then $5 \times 3 + 12 = 27$ but $27 \neq 62$ so this assumption is wrong!
27 is less than 62 so our guess $x = 3$ is too small.

Guess 2: This time, let's try a number bigger than 3 like 20.
Assume $x = 20$, then $5 \times 20 + 12 = 112$ but $112 \neq 62$ so again we are wrong!
112 is bigger than 62 so our guess $x = 20$ is too big.

Guess 3: This time, let's try a number between 3 and 20 because 3 is too small and 20 too big.
Assume x = 10, then $5 \times 10 + 12 = 62$ and this time we are correct ☺.

We say that $x = 10$ satisfies the equation $5x + 12 = 62$.

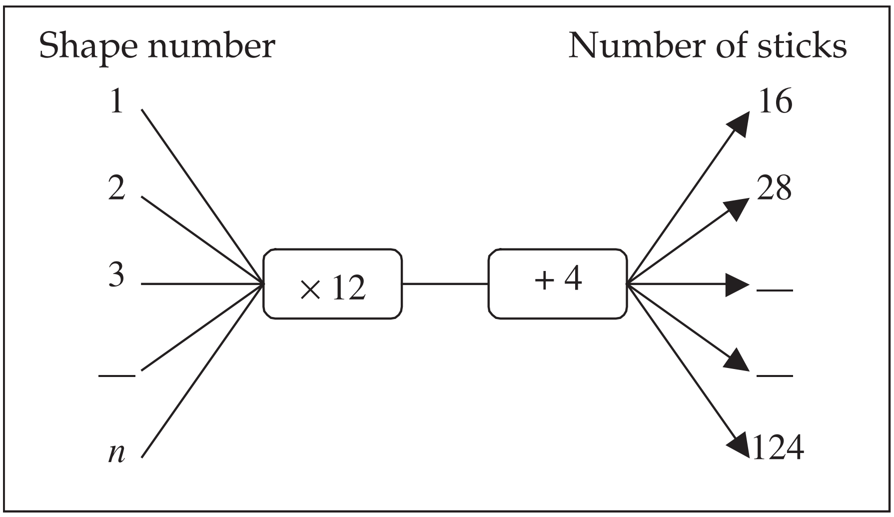
Time needed
15 minutes

Activity 7:
Solving equations using trial and improvement

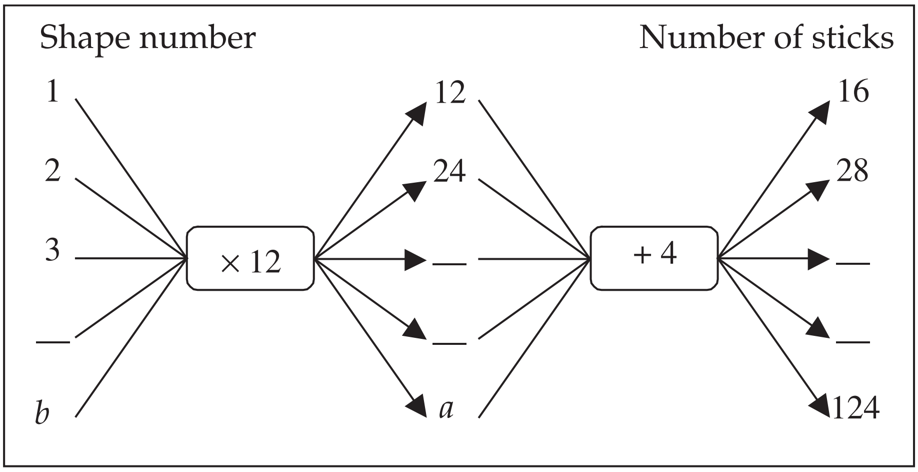
1. Solve the following equations using trial and improvement
   (a) $x + 9 = 17$
   (b) $3a + 5 = 125$
   (c) $2y + 7 = 39$
   (c) $5b + 8 = 108$
   (e) $4x - 12 = 36$
   (d) $19 - 2a = 7$
   (g) $7 = 21 - 4y$
   (h) $2x + 3 = 3$

Solving equations using flow diagrams
You can also use flow diagrams to solve equations.

Thandiswa could show her rule like this. Remember her rule is $12n + 4$, where n represents the shape number.



To solve the equation $12n + 4 = 124$ you find the value for n in the bottom line of the flow diagram. In other words find the input value that will give a particular output value.



Think about what happens to the input number at each stage of the flow diagram as illustrated above.

Time needed
25 minutes

Activity 8:
Solving equations using flow diagrams

1. Draw a flow diagram for each of the following rules.
   (a) $3n + 2$ (b) $4 + 8n$
   (c) $5n - 3$

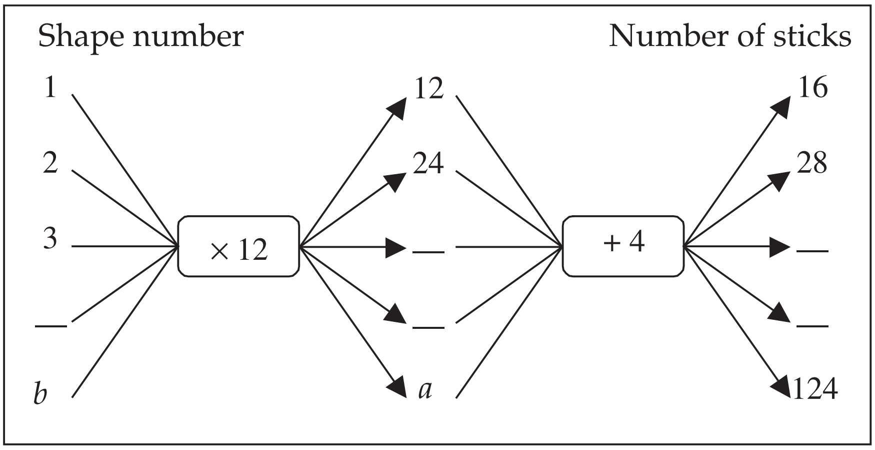
2. Use the flow diagrams you have drawn to solve the following equations:
   (a) $3n + 2 = 17$ (b) $3n + 2 = 65$
   (c) $4 + 8n = 36$ (d) $4 + 8n = 100$
   (e) $5n - 3 = 27$ (f) $5n - 3 = 42$

3. Describe, in words, the steps you used to solve the equations in question 2.
4. Compare your answers with a partner.

What have you learned?

Solving equations by isolating the unknown

In the "solving the equation" activity above you realised that we were effectively "undoing" the steps of the original flow diagram to find the value of the unknown.



Consider the input number 2. After the first operator (×12) its value had changed to 2 × 12 = 24.

24 then became the input number for the second operator (+4) and its value changed to 24 + 4 = 28.

To solve the equation $12n + 4 = 28$ is to find the input number that gives an output number of 28 (i.e. $n = 2$).

In order to "undo" the flow diagram's step for $12n + 4 = 28$ begin by "undoing" the +4. You do this by subtracting 4 and this takes you back to the number 24.

You could write this as follows:

$$12n + 4 = 28$$
$$12n + 4 - 4 = 28 - 4$$
$$12n = 24$$

Then you "undo" the flow diagram's step for 12 × n = 24. You do this by "undoing" the by dividing by 12 and this takes you right back to the number 2 which we started with.

You could write this as follows:

$$12n = 24$$
$$12n \div 12 = 24 \div 12$$
$$n = 2$$

Let's return to the equation we started with: $12n = 124$ and repeat the above process.

First "undo" the + 4:

$12n + 4 = 124$
$12n + 4 - 4 = 124 - 4$
$12n = 120$

Next "undo" the × 12:

$12n = 120$
$12n \div 12 = 120 \div 12$
$n = 10$

With practice, you might not write out all these steps but simply write:

	$12n + 4 = 124$
subtract 4 and:	$12n = 120$
divide by 12 and:	$n = 10$

or even just…

$12n + 4 = 124$
$\therefore\ 12n = 120$
$\therefore\ n = 10$

Time needed
30 minutes

Activity 9:
Practising solving equations by isolating the unknown

1. Solve each of the following equations by the method of isolating the unknown. You may find it easier to first draw a flow diagram to identify the steps.

(a) $6a - 4 = 8$
(b) $4m + 60 = 120$
(c) $2b - 5 = 11$
(d) $4a - 32 = 0$
(e) $-24 + 7x = 4$
(f) $12p = 108$
(g) $25 - 3x = 7$
(h) $3y + 2 = 23$
(i) $2 + 15n = 137$
(j) $-45\frac{1}{2} + 21y = 17\frac{1}{2}$

7. Solving inequalities

Let's return to the Thandiswa's problem. She wanted to find which shape in the pattern she can make using exactly 124 sticks. By solving the equation $12n + 4 = 124$ and getting $n = 10$ we decided that she cn make the 10^{th} shape with this many sticks.

Maybe you also realised that she can make any of the first ten shapes if she has 124 sticks but in most cases she would have some sticks left over.

There is a different kind of problem – we call it an inequality. With inequalities there is more than one number that will satisfy the equation. With inequalities we want to find all the values of the variable that satisfy the equation.

The problem becomes: "Which shapes can Thandiswa make if she had 124 sticks". The answer is that Thandiswa can make any of the first 10 shapes. You write the inequality like this $12n + 4 \leq 124$. The sign $\leq$ means "less than or equal to". The solution to the inequality is $n \leq 10$.

If our problem was: "Which shapes can Thandiswa make if she has to use all 124 sticks or more," the answer will be that she can make the 10^{th} shape or any shape bigger than that. You write that inequality like this $12n + 4 \geq 124$. The sign ≥ 124 means "greater than or equal to". The solution to the inequality is $n \geq 10$.

Time needed
30 minutes

Activity 10:
Solving inequalities

Write this activity on separate paper and put it in your portfolio.

1. Solve each of the following problems.
   a. Palesa's vetkoek:
      What are all the different numbers of vetkoek that somebody can buy if they have R24.00? Write an inequality to describe this example.

   b. Helen's sunflower:
      For which days after Helen brought the sunflower to school was the sunflower plant shorter than 27cm? Write an inequality to describe this example.

   c. Sunny Days Crèche's electricity:
      On which days of the month would you expect the balance of units of electricity to be less than 540? Write an inequality to describe this example.

8. Expressions involving brackets

Remember Thandiswa and Richard both made a rule for the pattern with sticks.
You said that $4 + 12n = 4 \times (3n + 1)$.
You showed it was true by completing a table of values.

You also realised that you needed to learn some algebraic techniques to show that this is true without using a table of values.

Now you are ready to try.

If Richard wants to show that his rule is the same as Thandiswa's rule he can change his rule to make it look the same as Thandiswa's rule. He can do this as follows:

$4 \times (3n + 1) = (3n + 1) + (3n + 1) + (3n + 1) + (3n + 1)$
$= 3n + 1 + 3n + 1 + 3n + 1 + 3n + 1$
$= 3n + 3n + 3n + 3n + 1 + 1 + 1 + 1$
$= 12n + 4$

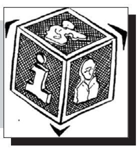
Look carefully at what this means:
$4 \times (3n + 1)$ is the same as adding four lots of (3n + 1). You saw this before when you saw that $4a = 4 \times a = a + a + a + a$.
In the second last step above, you had 4 lots of $3n$ and 4 lots of 1. So you ended up with 4 lots of each of the terms in the brackets. With practice you may simply write:
$4 \times (3n + 1) = 4 \;\; 3n + 4 \times 1$
$= 12n + 4$

Richard simply "removed the brackets".

If Thandiswa wants to show that her rule is the same as Richards's rule she will "introduce brackets" into her rule. She can do this as follows:

$12n + 4 = 4 \times 3n + 4 \times 1$
$= 4 \times (3n + 1)$

Thandiswa realised that both $12n$ and 4 can be written as 4× something. She realised that she can group the things in a bracket and put 4× in front of this bracket.



9. Factorising

The sentence above is a very clumsy way of describing what mathematicians call factorising an expression.

When you factorise a number you follow steps to break the number up into two or more numbers which can be multiplied together to make the original number. So, for example, 5 and 7 are factors of 35 because $7 \times 5 = 35$. Also 2 and 3 and 7 are factors of 42 because $2 \times 3 \times 7 = 42$.

So, when you factorise an algebraic expression you follow the steps to break up the expression into two or more expressions which can be multiplied together to make the original expression. These expressions are called factors of the original expression. For example:
$6n = 2 \times 3 \times \mathrm{n}$
$6n + 12m = 3 \times (2n + 4m)$
$8x^2 = 8 \times x \times x$
$8x^2 + 7x = x(8x + 7)$

Sometimes there is more than one possibility when factorising. E.g. $8x^2 + 6x$ could be factorised as $2 \times (4x^2 + 3x)$ or as $x \times (8x + 6)$ or as $2x \times (4x + 3)$. We always try ensure that the terms in the brackets have no factors in common. Therefore, we don't choose $2 \times (4x^2 + 3x)$ because here the terms in the expression in the bracket still have an x in common and we don't choose $x \times (8x + 6)$ because here the terms in the expression in the bracket still have a 2 in common.
$8x^2 + 6x = 2x \times (4x + 3) = 2x(4x + 3)$ is the best factorisation because $4x$ and +3 do not have a factor in common.

This factorisation technique is called the "common factor" technique.

Time needed
30 minutes

Activity 11:
Factorising using the "common factor" technique

1. Factorise each of the following expressions:

(a)	$3a + 6$	(b)	$5x - 45$
(c)	$2y - 16$	(d)	$5a + 15$
(e)	$35 - 7p$	(f)	$a^2 - 3a$
(g)	$n^2 - 5n$	(h)	$t^2 + t$
(i)	$2x^2 - x$	(j)	$x^2 + 2x$
(k)	$6y + 8$	(l)	$15n + 20$
(m)	$9p - 21$	(n)	$6a^2 + 9a$
(o)	$8x - 24x^2$	(p)	$10y^2 + 15y$
(q)	$12m^2 - 4m$	(r)	$2ab + 3b$
(s)	$6x + 18xy$	(t)	$3pq - 9p$

2. Compare your answers to questions in Activity 11 with a partner.

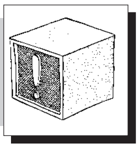
Factorising using the "difference of squares" technique
Betsy and Vuyo have to find a rule for each of the patterns in the table below:

Term Number	1	2	3	4	5	6
Pattern 1	0	1	2	3	4	5
Pattern 2	2	3	4	5	6	7
Pattern 3	0	3	8	15	24	35

Vuyo and Betsy developed the following rules for pattern 3 in the table above:

Vuyo's rule is $(n - 1)(n + 1)$. She explains her rule as follows: I noticed that pattern 1 could have the rule $n - 1$ and pattern 2 could have the rule $n + 1$. The numbers in Pattern 3 are the product of the numbers in Pattern 1 and Pattern 2.

Betsy's rule for pattern 3 is $\boxed{n^2 - 1}$. She explains her rule as follows: I noticed that $1^2 = 1$, $2^2 = 4$, $3^2 = 9$ etc. The numbers in pattern 3 are all one less than these square numbers.



Stop and Think

How can Vuyo and Betsy show each other that both of their rules are correct and are the same? In other words how can Vuyo and Betsy show each other that $(n - 1)(n + 1) = n^2 - 1$?

You have realised that Vuyo and Betsy can use a table of values to show each other. But they can also use algebraic techniques. For Vuyo this means "removing the brackets" and for Betsy this means "putting in some brackets".

To remove the brackets Vuyo uses a similar approach to Richard:

$(n - 1)(n + 1) = (n - 1) \times n + (n - 1) \times 1$ *Vuyo is treating n + 1 in he same way that Richard used 4.*

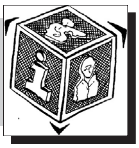
$= n \times (n - 1) + 1 \times (n - 1)$

$= n \times n - n \times 1 + 1 \times n - 1 \times 1$

$= n^2 - n + n - 1$

$= n^2 - 1$

Whew! That was a lot of work!



The difference of squares

For Betsy to "put some brackets" into her rule (i.e. to factorise the expression) she needs to know an algebraic technique called the "difference of squares".

The difference of squares technique: $\boxed{a^2 - b^2 = (a - b)(a + b)}$

The word difference means subtraction. So you can only use the "difference of squares" technique when the two terms are separated by a minus sign, like $a^2 - b^2$. You cannot use it when the terms are separated by a plus sign like $a^2 + b^2$.

Betsy can write $n^2 - 1$ as $n^2 - 1^2$. So $n^2 - 1^2 = (n - 1)(n + 1)$.

Sometimes in Mathematics you have to just learn a technique which others have already proved is true. The difference of squares is an example of this.

Look at some more examples of this technique:

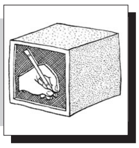
$25 - x^2 = 5^2 - x^2$

$= (5 - x)(5 + x)$

$p^2 - 9q^2 = p^2 - (3q)^2$

$= (p - 3q)(p + 3q)$

Time needed
30 minutes

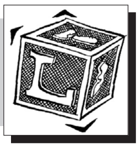


Activity 12:
Factorising using the "difference of squares" technique

Do this activity on a separate paper and put it into your portfolio.

1. Factorise each of the following expressions:

(a) $a^2 - 4$	(b) $9 - y^2$
(c) $b^2 - 16$	(d) $49 - n^2$
(e) $x^2 - 1$	(f) $a^2 - 64$
(g) $n^2 - 100$	(h) $81 - p^2$
(i) $4x^2 - 9$	(j) $x^2 - 25y^2$



What have you learned?

Let's summarise what you have learned so far in this unit.

- There are at least two ways of showing that rules are equivalent:
  - o by completing a table of values (substitution)
  - o by using algebraic techniques

- Terms in an expression are separated by plus and minus signs.
- Terms which have the same variables are called like terms
- Terms which do not have the same variables are called unlike terms
- To simplify algebraic expressions you add or subtract the like terms.
- An exponent is a little number on the upper right of a variable to show repeated multiplication like $x^3 = x \times x \times x$
- If the exponent is a 1, then we leave it out (e.g. $x^1 = x$)
- We can only simplify terms with exponents if the bases are the same.
- When you multiply expressions with like bases, you add their exponents: $x^a \times x^b = x^{a+b}$
- When you divide expressions with like bases, you subtract their exponents: $x^a \surd x^b = x^{a-b}$
- There are at least two ways of "solving an equation":
  - o To solve equations by TRIAL & IMPROVEMENT you "guess" a solution and then check to see if it satisfies the equation. If your original guess does not satisfy the equation you choose a new "improved" guess and check it. You carry on in this way until you find the solution.
  - o To solve an equation by ISOLATING THE UNKNOWN we effectively "undoing" the steps of the equation (which can be represented by means of a flow diagram) to find the value of the unknown.
- To solve an inequality you find the values in the table of values which satisfy the given condition. We can write an inequality to describe any situation where the variable can be described in the following ways "bigger/greater than something" (>), "less than something" (<), "bigger/greater than and/or equal to something" (≥), "less than and/or equal to something" (≤).

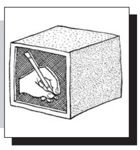
- You can simplify expressions if you "remove" brackets. In expressions involving brackets by writing the expression in full and simplifying the like terms. For example:

  $3 \times (2x + 5) = (2x + 5) + (2x + 5) + (2x + 5)$
  $= 2x + 5 + 2x + 5 + 2x + 5$
  $= 2x + 2x + 2x = 5 + 5 + 5$
  $= 6x + 15$

  With practice you will see that:
  $3 \times (2x + 5) = 3 \times 2x + 3 \times 5$
  $= 6x + 15$

- Two algebraic techniques for factorising an expression are:
  - o The "common factor" technique where you break an expression up into two or more expressions like this
  - o The "difference of squares technique" when two "squared" terms are separated by a minus sign:

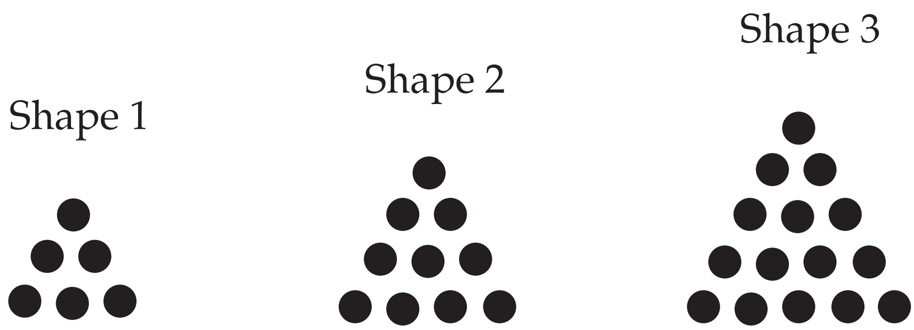
Time needed 1 hour



Activity 13:
Assess yourself

Write this activity on separate paper and put it in your portfolio.

1. Study the diagram below and then copy and complete the table which follows



Shape number	1	2	3	4	5	6
Number of circles	6	10				

2. Develop a rule for finding the number of circles for any given shape number.
3. Tanya and Jeremy have the following rules for finding the number of circles. Complete the table below to show that both rules are correct

Tanya's rule: $4n + 2$ Jeremy's rule: $2(2n + 1)$

Shape number	1	2	3	4	5	6
Number of sticks	6	8				
Tanya's rule						
Jeremy's rule						

4. Show, by "removing the brackets" that Jeremy's rule is the same as Tanya's rule.
5. Show by factorising by means of the "common factor" technique that Tanya's rule is the same as Jeremy's rule.
6. Give the number of terms for each of the following expressions:
   (a) $6x^2 + 4x - b$ (b) $3x \times 2a$
   (c) $6 \times b - 4 \times a^2 + 2$ (d) $2ab - 4a$
   (e) $\frac{1}{2}x + 3y \times 5$ (f) $\frac{x}{3y} + 2y + 3x - 5$

7. State whether the terms in the expression below are like or unlike:
   (a) $x + 2x$ (b) $4y \times 3b$
   (c) $3p + p$ (d) $16 - 3x$
   (e) $3 - 20$ (f) $3a^2 + 4a^2$
   (g) $3a^2 + 4a$ (h) $2ab + 5a$

8. In each of the expression below, two of are the terms are like. Identify the like terms.
   (a) $6ab - 3b + 2ab$ (b) $6x^2 + x - 5 + 3x$
   (c) $3a^2 - 6a + 2 + 10a^2b + 5$ (d) $a + 3 + b - 6$
   (e) $2abc + a^2b - 3bc + 6a^2b$

9. Simplify the following by adding or subtracting the like terms.
   (a) $3x + 6x$ (b) $10y \times y$
   (c) $3a + 2a$ (d) $4p^2 - p^2$
   (e) $3x + 2x - 5x$ (f) $6y + 2x + 6y$

10. Simplify each of the following expressions:
   (a) $x^{10} \times x^4$ (b) $a \times a$
   (c) $p^2 \times p$ (d) b2 × b5 × b3
   (e) $x^{10} \div x^3$ (f) $y^5 \div y^4$
   (g) $a^6 \div a^2$ (h) $p^4 \div p^3$

11. Solve the following equations using trial and improvement
   (a) $2x + 5 = 11$ (b) $3a + 16 = 40$
   (c) $2y - 7 = 35$ (d) $4b + 8 = 56$
   (e) $8x - 12 = 188$

12. Draw a flow diagram for each of the following rules.
   (a) $4n + 2$ (b) $6 + 5n$
   (c) $2x - 3$

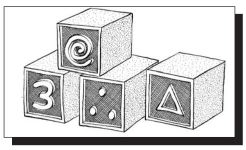
13. Using the flow diagrams you have drawn, solve the following equations:
   (a) $4n + 2 = 34$ (b) $4n + 2 = 122$
   (c) $6 + 5n = 41$ (d) $6 + 5n = 146$
   (e) $2x - 3 = 127$ (f) $2x - 3 = 999$

14. Solve each of the following equations by the method of isolating the unknown. Note: you may find it easier to first draw a flow diagram to identify the sequence of operations.
   (a) $3a - 10 = 35$ (b) $6a + 12 = 54$
   (c) $4n - 16 = 112$ (d) $31 - 3x = 25$
   (e) $2b - 32 = 0$

15. Refer to the table and shapes in question 1 of this Activity:
   a. What are all the different shape numbers that someone could make if they had less than 26 circles? Write an inequality to describe this situation.

16. Factorise each of the following expressions:
   (a) $2x + 4$ (b) $5y - 50$
   (c) $2a - 36$ (d) $6x - 18$
   (e) $20 - 2p$ (f) $x^2 - 2x$
   (g) $n^2 - n$ (h) $4t^2 + 2t$
   (i) $15x^2 + 45x$ (j) $6y^2 + 9y$

17. Factorise each of the following expressions:
   (a) $p^2 + q^2$ (b) $y^2 - 4$
   (c) $16 - y^2$ (d) $n^2 - 144$
   (f) $9a^2 - 49$



Linking patterns with your work with children

You can encourage even very young children to think algebraically. In the first Unit you saw ideas for activities where learners make up their own patterns, extend patterns that are started for them and try and find a "rule" in their own language to explain a pattern.

Here are two ideas for young children to start thinking about number in algebraic terms. They try to find the value of the unknown quantity (x).

1. **More, less, the same as**
   In a table with three columns you draw a number of faces. In the third column children draw more or less faces.

The same	☺ ☺	☺ ☺
One less	☺ ☺	☺
One more	☺ ☺	☺ ☺ ☺
Two more	☺ ☺	☺ ☺ ☺ ☺

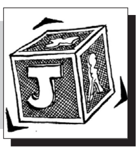
2. **Modelling a number story**
   Make up a simple number story. For example: Mama Lerato had 2 chickens she went to market and now she has 5. Can we find out how many she bought?

   Use pictures in a diagram or use real objects to show the problem:

   ☺ ☺ and [?] make ☺ ☺ ☺ ☺

   Let the children count and draw and discuss until they can tell you that you need to add 2 more chickens to make 4. You can ask the question like this: "What must I add to 2 chickens to make 4 chickens?"

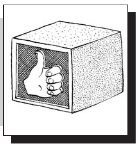
   Notice that in algebraic terms this is $2 + x = 4$. Of course you will not introduce this symbolic notation to the children. But the discussions and language you use can help to develop them to first identify and then find the value of the unknown, which is a way of thinking algebraically.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about the difference between like and unlike terms?
b. What did you learn from this unit about simplifying terms?
c. Write down one or two questions that you still have about solving equations and inequalities.
d. Write down one or two questions you still have about factorising expressions.



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Identifying the difference between like and unlike terms				
2. Gathering and adding/subtracting like terms				
3. Using the meaning of exponents				
4. Simplifying terms that involve both multiplication and division with exponents				
5. Solve equations using the “trial and improvement” technique				
6. Solve equations using the “isolating the unknown” technique (together with flow diagrams)				
7. Solve simple inequalities using tables of values				
8. Factorise expressions in order to show equivalence of expressions using both the “common factor” and “difference of squares” techniques				